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In $\Delta ABC, A, B, C$ – angles. Find:

$$\Omega(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \begin{vmatrix} \sin A & \sin B & \sin C \\ \sin(A+kx) & \sin(B+kx) & \sin(C+kx) \\ \cos(A+kx) & \cos(B+kx) & \cos(C+kx) \end{vmatrix}, x \in \mathbb{R}$$

*Proposed by Daniel Sitaru – Romania*

*Solution by Ravi Prakash-New Delhi-India*

$$\text{Let } \Delta(k,x) = \begin{vmatrix} \sin A & \sin B & \sin C \\ \sin(A+kx) & \sin(B+kx) & \sin(C+kx) \\ \cos(A+kx) & \cos(B+kx) & \cos(C+kx) \end{vmatrix}$$

*Using* $R_2 \to R_2 - (\cos kx)R_1;\ R_3 \to R_3 + (\sin kx)R_1$, *we get*

$$\Delta(k,x) = \begin{vmatrix} \sin A & \sin B & \sin C \\ \cos A \sin kx & \cos B \sin kx & \cos C \sin kx \\ \cos A \cos kx & \cos B \cos kx & \cos C \cos kn \end{vmatrix} = 0$$

*[*$\because R_2$ *and* $R_3$ *are proportional]. Now,*

$$\sum_{k=1}^{n} \Delta(k,x) = 0 \Rightarrow \Omega(x) = \lim_{n\to\infty} \sum_{k=1}^{n} \Delta(k,x) = 0$$