

R M M

ROMANIAN MATHEMATICAL MAGAZINE
www.ssmrmh.ro

If $x, y, z \in \left(0, \frac{\pi}{2}\right)$ then:

$$\prod \ln(1 + \tan^2 x) \cdot \prod \ln(1 + \cot^2 y) \leq \prod \ln^2 \left(\frac{2}{\sin 2z} \right)$$

Proposed by Daniel Sitaru – Romania

Solution by Ravi Prakash-New Delhi-India

$$\begin{aligned} \text{For } 0 < x < \frac{\pi}{2}; \ln(1 + \tan^2 x) \ln(1 + \cot^2 x) &\leq \\ \leq \left\{ \frac{\ln(1 + \tan^2 x) + \ln(1 + \cot^2 x)}{2} \right\}^2 &= \left\{ \frac{1}{2} \ln(\sec^2 x \csc^2 x) \right\}^2 = \\ &= \left(\ln \left(\frac{2}{\sin 2x} \right) \right)^2 \end{aligned}$$

$$\begin{aligned} \text{Now, } 0 < x, y, z < \frac{\pi}{2}; \prod \ln(1 + \tan^2 x) \prod \ln(1 + \cot^2 y) &= \\ = \prod \ln(1 + \tan^2 x) (1 + \cot^2 x) &\leq \prod \left[\ln \left(\frac{2}{\sin 2x} \right) \right]^2 \end{aligned}$$