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If $a,b,c\geq 0$ then:

$$e^{a+b+c}\leq (1+a)^{1+a}(1+b)^{1+b}(1+c)^{1+c}$$

Proposed by Daniel Sitaru-Romania

Solution 1 by Abdelhak Maoukouf-Casablanca-Morocco, Solution 2 by Soumitra Mandal-Chandar Nagore-India, Solution 3 by Kunihiko Chikaya-Tokyo-Japan, Solution 4 by Lazaros Zachariades-Thessaloniki-Greece

Solution 1 by Abdelhak Maoukouf-Casablanca-Morocco

$$\forall: 0<t\leq 1\ \ e^{t-1}\geq t\ \Leftrightarrow\ e^{1-t}\leq\frac{1}{t}$$

$$t=\frac{1}{x+1};x\geq 0\ \Rightarrow\ e^{\frac{x}{x+1}}\leq x+1\Leftrightarrow e^{x}\leq (x+1)^{x+1}$$

$$\Rightarrow\prod e^{a}\leq\prod(a+1)^{a+1}\Leftrightarrow\boxed{e^{\sum a}\leq\prod(a+1)^{a+1}}$$

Solution 2 by Soumitra Mandal-Chandar Nagore-India

We know that:

$$\boldsymbol{ln(1+x)\geq\frac{x}{x+1}\ ,x\geq 0\rightarrow(1+x)ln(1+x)\geq x}$$

$$\boldsymbol{\sum(1+a)ln(1+a)\geq\sum a}$$

$$\boldsymbol{\sum ln(1+a)^{1+a}\geq\sum a\rightarrow\prod\prod(1+a)^{1+a}\geq e^{\sum a}}$$

$$\boldsymbol{e^{a+b+c}\leq(1+a)^{1+a}(1+b)^{1+b}(1+c)^{1+c}}$$

Solution 3 by Kunihiko Chikaya-Tokyo-Japan

$$\boldsymbol{0\leq x\leq a\rightarrow 1\leq 1+x\leq 1+a\rightarrow ln(1+x)\geq 0}$$

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$$0 \leq \int_0^a ln(1+x)dx = (1+a)ln(1+a) - \int_0^a (1+x)\cdot\frac{1}{1+x}dx$$

$$0 \leq (1+a)ln(1+a) - a \rightarrow e^a \leq (1+a)^{1+a}, a \geq 0$$

$$\begin{cases} e^a \leq (1+a)^{1+a} \\ e^b \leq (1+b)^{1+b} \\ e^c \leq (1+c)^{1+c} \end{cases} \rightarrow e^{a+b+c} \leq (1+a)^{1+a}(1+b)^{1+b}(1+c)^{1+c}$$

Solution 4 by Lazaros Zachariades-Thessaloniki-Greece

$$\textit{It's } lnx \geq \frac{x-1}{x}, \forall x > 0 \textit{ thus } ln(1+x) \geq \frac{x}{x+1}$$

$$\textit{So} \begin{cases} (a+1)ln(a+1) \geq a \\ (b+1)ln(b+1) \geq b \\ (c+1)ln(c+1) \geq c \end{cases} \rightarrow \begin{cases} ln(a+1)^{a+1} \geq a \\ ln(b+1)^{b+1} \geq b \\ ln(c+1)^{c+1} \geq c \end{cases} \rightarrow \begin{cases} (a+1)^{a+1} \geq e^a \\ (b+1)^{b+1} \geq e^b \\ (c+1)^{c+1} \geq e^c \end{cases}$$

$$e^{a+b+c} \leq (1+a)^{1+a}(1+b)^{1+b}(1+c)^{1+c}$$