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ROMANIAN MATHEMATICAL MAGAZINE
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If $0 < a, b, c \leq 1$ ***then:***

$$4\cdot a^{\frac{2a}{a+b}}\cdot b^{\frac{2b}{a+b}}+4\cdot b^{\frac{2b}{b+c}}\cdot c^{\frac{2c}{b+c}}+4\cdot c^{\frac{2c}{c+a}}\cdot a^{\frac{2a}{c+a}}\leq 9+(2a-b)^2+(2b-c)^2+(2c-a)^2$$

Proposed by Daniel Sitaru – Romania

Solution by Soumava Chakraborty-Kolkata-India

$$\sqrt[a+b]{(a^2)^a\cdot(b^2)^b}\overset{AM-GM}{\leq}\frac{a^2\cdot a+b^2\cdot b}{a+b}=a^2-ab+b^2$$

$$4\cdot\sqrt[a+b]{(a^2)^a\cdot(b^2)^b}\leq 4(a^2-ab+b^2)=(2a-b)^2+3b^2$$

$$\sum 4\cdot\sqrt[a+b]{(a^2)^a\cdot(b^2)^b}=\sum(2a-b)^2+3\sum b^2\leq$$

$$\leq\sum(2a-b)^2+3$$

$$4\cdot a^{\frac{2a}{a+b}}\cdot b^{\frac{2b}{a+b}}+4\cdot b^{\frac{2b}{b+c}}\cdot c^{\frac{2c}{b+c}}+4\cdot c^{\frac{2c}{c+a}}\cdot a^{\frac{2a}{c+a}}\leq\sum(2a-b)^2+3$$