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If $x, y, z \in \mathbb{R}$ then :

$$\prod\left(x^4 - x^3 + \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}\right) < \prod(2x^4 - 2x^3 + 2x^2 - 2x + 2)$$

Proposed by Daniel Sitaru-Romania

Solution by Ravi Prakash-New Delhi-India

Consider:

$$2x^4 - 2x^3 + 2x^2 - 2x + 2 - \left(x^4 - x^3 + \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}\right) =$$

$$= x^4 - x^3 + \frac{3}{2}x^2 - \frac{5}{2}x + \frac{3}{2} = f(x)$$

For $x \le 0, f(x) > 0$

For $x \ge 1, f(x) = x^3(x-1) + \frac{1}{6}(9x^2 - 15x + 9) =$

$$= x^3(x-1) + \frac{1}{6}(3x-3)^2 + \frac{1}{2}x > 0$$

For $0 < x < 1, f(x) = x^4 - x^3 + \frac{3}{2}x^2 - \frac{5}{2}x + \frac{3}{2} =$

$$= x^2\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}x^2 - \frac{5}{2}x + \frac{3}{2} = x^2\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}(x-1)^2 + \frac{1}{4} > 0$$

$$f(x) > 0 \to 2x^4 - 2x^3 + 2x^2 - 2x + 2 > x^4 - x^3 + \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}$$

$$\prod\left(x^4 - x^3 + \frac{1}{2}x^2 + \frac{1}{2}x + \frac{1}{2}\right) < \prod(2x^4 - 2x^3 + 2x^2 - 2x + 2)$$