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If $a, b, c > 0, a^4 + b^4 + c^4 = 3$ then:

$$\frac{a^2 b^4}{a^{18} + 8} + \frac{b^2 c^4}{b^{18} + 8} + \frac{c^2 a^4}{c^{18} + 8} \leq \frac{1}{3}$$

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Solution by Lazaros Zachariades-Thessaloniki-Greece

$$\begin{aligned} a^{18} + 8 &= a^{18} + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 \stackrel{AM-GM}{\geq} 9\sqrt[9]{a^{18}} \\ a^{18} + 8 &\geq 9a^2 \rightarrow \frac{1}{a^{18} + 8} \leq \frac{1}{9a^2} \rightarrow \frac{a^2 b^4}{a^{18} + 8} \leq \frac{b^4}{9} \\ \sum \frac{a^2 b^4}{a^{18} + 8} &\leq \sum \frac{b^4}{9} = \frac{1}{9}(a^4 + b^4 + c^4) = \frac{3}{9} = \frac{1}{3} \end{aligned}$$