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If $a,b,c>0, a^2+b^2+c^2=3$ then:

$$\sum(a+b)\sqrt{8(a^2+b^2)}\le 18+2(ab+bc+ca)$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Dimitris Kastriotis-Athens-Greece

Solution 2 by Soumava Chakraborty-Kolkata-India

Solution 1 by Dimitris Kastriotis-Athens-Greece

$$(a+b)\sqrt{8(a^2+b^2)}=\sqrt{4(a^2+b^2)\cdot 2(a+b)^2}\overset{AM-GM}{\le}$$

$$\le\frac{4(a^2+b^2)+2(a+b)^2}{2}=3a^2+3b^2+2ab$$

$$\sum(a+b)\sqrt{8(a^2+b^2)}\le 6\sum a^2+2\sum ab=6\cdot 3+2\sum ab$$

$$\sum(a+b)\sqrt{8(a^2+b^2)}\le 18+2(ab+bc+ca)$$

Solution 2 by Soumava Chakraborty-Kolkata-India

$$\sum(a+b)\sqrt{8(a^2+b^2)}\overset{CBS}{\le}\sqrt{\sum(a+b)^2}\cdot\sqrt{8\sum(a^2+b^2)}=$$

$$=\sqrt{2\sum ab+2\sum a^2}\cdot\sqrt{8(3+3)}=\sqrt{(2x+6)\cdot 48}\le 18+2x\ (to\ prove)$$

$$\sum ab=x$$

$$\sqrt{(2x+6)\cdot 48}\le 18+2x\leftrightarrow(x+9)^2\ge 24(x+3)\leftrightarrow(x-3)^2\ge 0$$