PP26169

MIHÁLY BENCZE

In all triangle ABC holds $3+\sum\cos(A-B)\geq\frac{12r}{R}$

Solution by Kevin Soto Palacios - Huarmey - Peru.
Probar en un triángulo ABC

$$1+\cos(A-B)+1+\cos(B-C)+1+\cos(C-A)\geq\frac{12r}{R}$$
$$\Leftrightarrow 2\cos^2\left(\frac{A-B}{2}\right)+2\cos^2\left(\frac{B-C}{2}\right)+2\cos^2\left(\frac{C-A}{2}\right)\geq\frac{12r}{R}$$
$$\Leftrightarrow \cos^2\left(\frac{A-B}{2}\right)+\cos^2\left(\frac{B-C}{2}\right)+\cos^2\left(\frac{C-A}{2}\right)\geq\frac{6r}{R}$$

Es suficiente probar

$$\cos^2\left(\frac{A-B}{2}\right)\geq\frac{2r}{R}=8\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=4\sin\frac{C}{2}\left(\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right)=$$
$$=4\sin\frac{C}{2}\left(\cos\left(\frac{A-B}{2}\right)-\sin\frac{C}{2}\right)$$

(A)

$$\Leftrightarrow\cos^2\left(\frac{A-B}{2}\right)-4\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+4\sin^2\frac{C}{2}=\left(\cos\left(\frac{A-B}{2}\right)-2\sin\frac{C}{2}\right)^2\geq 0$$

Análogamente para los siguientes términos

(B) $$\cos^2\left(\frac{B-C}{2}\right)\geq\frac{2r}{R}$$

(C) $$\cos^2\left(\frac{C-A}{2}\right)\geq\frac{2r}{R}$$

Sumando (A) + (B) + (C)

$$\Rightarrow\cos^2\left(\frac{A-B}{2}\right)+\cos^2\left(\frac{B-C}{2}\right)+\cos^2\left(\frac{C-A}{2}\right)\geq\frac{2r}{R}+\frac{2r}{R}+\frac{2r}{R}=\frac{6r}{R}$$

(LQQD)

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