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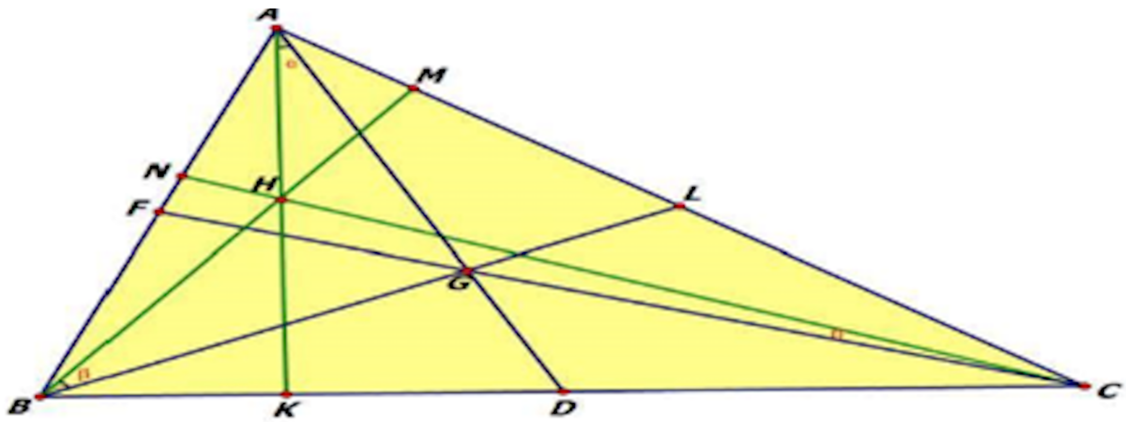
If in ΔABC acute, $a > b > c$, H – orthocenter, G - centroid

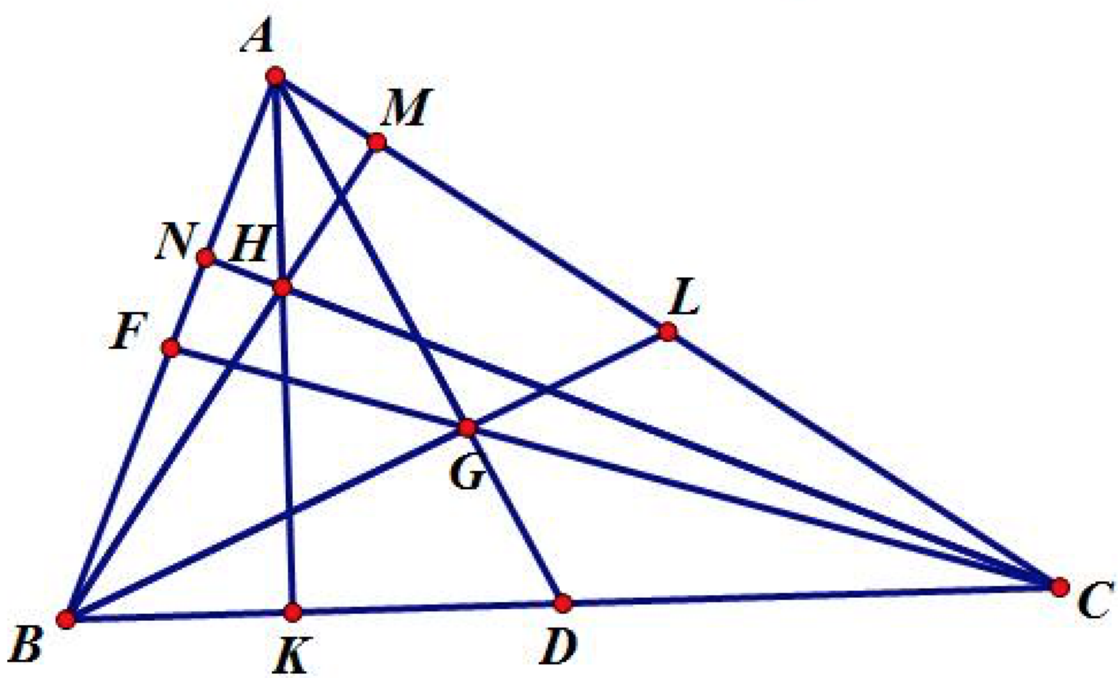
$m(\sphericalangle HAG) = \alpha, m(\sphericalangle HBG) = \beta, m(\sphericalangle HCG) = \theta$ then:

$$\tan\beta = \tan\alpha + \tan\theta$$



Proposed by Mehmet Sahin-Ankara-Turkey

Solution by Khanh Hung Vu-Ho Chi Minh-Vietnam



If in ΔABC – acute, $a > b > c$, H - orthocenter, G - centroid,

$\angle HAG = \alpha, \angle HBG = \beta, \angle HCG = \gamma$ *then*

$$\tan\beta = \tan\alpha + \tan\gamma$$

We have $h_b = \frac{2S}{b} \Rightarrow h_b^2 = \frac{4S^2}{b^2} = \frac{4\cdot\frac{2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)}{16}}{b^2} =$

$$= \frac{2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)}{4b^2}$$

We have $\tan\beta = \frac{ML}{BM} = \frac{\sqrt{m_a^2-h_b^2}}{h)b} = \frac{\sqrt{\frac{2a^2+2c^2-b^2}{4}-\frac{2(a^2b^2+b^2c^2+c^2a^2)-(a^4+b^4+c^4)}{4b^2}}}{\frac{2S}{b}} =$

$$= \frac{\sqrt{\frac{(2a^2+2c^2-b^2)\cdot b^2-2(a^2b^2+b^2c^2+c^2a^2)+(a^4+b^4+c^4)}{4b^2}}}{\frac{2S}{b}}$$

$\Rightarrow \tan\beta = \frac{\sqrt{\frac{a^4+c^4-2a^2c^2}{4b^2}}}{\frac{2S}{b}} = \frac{\frac{a^2-c^2}{2b}}{\frac{2S}{b}} = \frac{a^2-c^2}{4S}$ ***(1) (Since*** $a > c$***)***

Similarly, we have $\tan\alpha = \frac{b^2-c^2}{4S}$ ***and*** $\tan\gamma = \frac{a^2-b^2}{4S}$

$\Rightarrow \tan\alpha + \tan\gamma = \frac{a^2-c^2}{4S}$ ***(2)***

(1) and (2) $\Rightarrow \tan\beta = \tan\alpha + \tan\gamma$

(QED)