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Find $x, y, z, t \in \mathbb{R}$ such that:

$$5x^2 + 5y^2 + 5z^2 + 5t^2 - 5xy - 5yz - 5zt - 5t + 2 = 0$$

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Solution by Subhajit Chattopadhyay-Bolpur-India

$$5x^2 + 5y^2 + 5z^2 + 5t^2 - 5xy - 5yz - 5zt - 5t + 2 = 0$$

$$\text{or, } 5\left(x - \frac{y}{2}\right)^2 + \frac{15y^2}{4} + 5z^2 + 5t^2 - 5yz - 5zt - 5t + 2 = 0$$

$$\text{or, } 5\left(x - \frac{y}{2}\right)^2 + 5\left(\frac{\sqrt{3}y}{2} - \frac{z}{\sqrt{3}}\right)^2 + \frac{10z^2}{3} - 5zt + 5t^2 - 5t + 2 = 0$$

$$\text{or, } 5\left(x - \frac{y}{2}\right)^2 + 5\left(\frac{\sqrt{3}y}{2} - \frac{z}{\sqrt{3}}\right)^2 + 5\left(\frac{\sqrt{2}z}{\sqrt{3}} - \frac{\sqrt{3}t}{2\sqrt{2}}\right)^2 + \frac{25t^2}{8} - 5t + 2 = 0$$

$$\text{or, } 5\left(x - \frac{y}{2}\right)^2 + 5\left(\frac{\sqrt{3}y}{2} - \frac{z}{\sqrt{3}}\right)^2 + 5\left(\frac{\sqrt{2}z}{\sqrt{3}} - \frac{\sqrt{3}t}{2\sqrt{2}}\right)^2 + \left(\frac{5t}{2\sqrt{2}} - \sqrt{2}\right)^2 = 0$$

$$t, x, y, z \in \mathbb{R} \Rightarrow$$

$$x = \frac{y}{2}; \frac{\sqrt{3}y}{2} = \frac{z}{\sqrt{3}}$$

$$\frac{\sqrt{2}z}{\sqrt{3}} = \frac{\sqrt{3}t}{2\sqrt{2}}; \frac{5t}{2\sqrt{2}} = \sqrt{2}$$

$$\Rightarrow t = \frac{4}{5}, z = \frac{3}{5}; y = \frac{2}{5}; x = \frac{1}{5}$$