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If $a,b,c>0, ab+bc+ca=\frac{1}{5}$ then:

$$\frac{a}{\sqrt[3]{2b+3c}}+\frac{b}{\sqrt[3]{2c+3a}}+\frac{c}{\sqrt[3]{2a+3b}}\geq(a+b+c)\sqrt[3]{a+b+c}$$

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Solution by Nguyen Minh Tri-Ho Chi Minh-Vietnam

By Holder, we have:

$$\left(\frac{a}{\sqrt[3]{2b+3c}}+\frac{b}{\sqrt[3]{2c+3a}}+\frac{c}{\sqrt[3]{2a+3b}}\right)^3[a(2b+3c)+b(2c+3a)+c(2a+3b)]\geq(a+b+c)^4$$

$$\Leftrightarrow\left(\frac{a}{\sqrt[3]{2b+3c}}+\frac{b}{\sqrt[3]{2c+3a}}+\frac{c}{\sqrt[3]{2a+3b}}\right)^3\cdot 5(ab+bc+ca)\geq(a+b+c)^4$$

$$\Leftrightarrow\left(\frac{a}{\sqrt[3]{2b+3c}}+\frac{b}{\sqrt[3]{2c+3a}}+\frac{c}{\sqrt[3]{2a+3b}}\right)^3\geq(a+b+c)^4\left(ab+bc+ca=\frac{1}{5}\right)$$

$$\Leftrightarrow\frac{a}{\sqrt[3]{2b+3c}}+\frac{b}{\sqrt[3]{2c+3a}}+\frac{c}{\sqrt[3]{2a+3b}}\geq(a+b+c)\sqrt[3]{a+b+c}$$

(QED)