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If $a, b, c \geq 1$ then:

$$\frac{(1+a)(1+b)(1+c)}{(1+3a)(1+3b)(1+3c)} \geq \frac{2}{8abc + (1+a)(1+b)(1+c)}$$

Proposed by Daniel Sitaru – Romania

Solution 1 by Kevin Soto Palacios – Huarmey – Peru

Solution 2 by Nguyen Minh Tri-Ho Chi Minh-Vietnam

Solution 1 by Kevin Soto Palacios – Huarmey – Peru

Siendo $a, b, c \geq 1$. Probar que

$$\frac{(1+a)(1+b)(1+c)}{(1+3a)(1+3b)(1+3c)} \geq \frac{2}{8abc + (1+a)(1+b)(1+c)}$$

Como $a, b, c \geq 1$, implica que $a, b, c \in \mathbb{R}^+$

Tener en cuenta lo siguiente

$$\frac{1+a}{1+3a} \geq \frac{1}{1+a} \Leftrightarrow (1+a)^2 - (1+3a) = a(a-1) \geq 0,$$

$$\frac{1+b}{1+3b} \geq \frac{1}{1+b}, \quad \frac{1+c}{1+3c} \geq \frac{1}{1+c}$$

Del mismo modo

$$\frac{2a}{1+a} \geq 1 \Leftrightarrow a \geq 1, \quad \frac{2b}{b+1} \geq 1, \quad \frac{2c}{c+1} \geq 1$$

Por consiguiente

$$\frac{(1+a)(1+b)(1+c)}{(1+3a)(1+3b)(1+3c)} \geq \frac{1}{(1+a)(1+b)(1+c)}$$

Es suficiente demostrar

$$\frac{1}{(1+a)(1+b)(1+c)} \geq \frac{2}{8abc + (1+a)(1+b)(1+c)} \Leftrightarrow$$

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$$\Leftrightarrow 1 + \frac{2a}{1+a} \cdot \frac{2b}{1+b} \cdot \frac{2c}{1+c} \geq 2 \quad (\text{Lo cual es cierto})$$

Solution 2 by Nguyen Minh Tri-Ho Chi Minh-Vietnam

Because $a, b, c \geq 1$, we have:

$$4a \geq 3a + 1$$

$$4b \geq 3b + 1$$

$$4c \geq 3c + 1$$

$$\Rightarrow 64abc \geq (1 + 3a)(1 + 3b)(1 + 3c)$$

$$\Rightarrow \frac{(1+a)(1+b)(1+c)}{(1+3a)(1+3b)(1+3c)} \geq \frac{(1+a)(1+b)(1+c)}{64abc} \quad (1)$$

Similarly we have:

$$2a \geq 1 + a$$

$$2b \geq 1 + b$$

$$2c \geq 1 + c$$

$$\Rightarrow 8abc \geq (1 + a)(1 + b)(1 + c)$$

$$\Rightarrow \frac{2}{8abc + (1+a)(1+b)(1+c)} \leq \frac{1}{(1+a)(1+b)(1+c)} \quad (2)$$

So we have to prove that:

$$\frac{(1+a)(1+b)(1+c)}{64abc} \geq \frac{1}{(1+a)(1+b)(1+c)} \quad (3)$$

$$\Leftrightarrow (1 + a)^2(1 + b)^2(1 + c)^2 \geq 64abc$$

$$\text{Have: } (1 + a)^2 \geq 4a; (1 + b)^2 \geq 4b; (1 + c)^2 \geq 4c$$

$$\Rightarrow (1 + a)^2(1 + b)^2(1 + c)^2 \geq 64abc \Rightarrow (3) \text{ true}$$

From (1), (2), (3) we have

$$\frac{(1 + a)(1 + b)(1 + c)}{(1 + 3a)(1 + 3b)(1 + 3c)} \geq \frac{2}{8abc + (1 + a)(1 + b)(1 + c)}$$

(Q.E.D.)