PROPOSED PROBLEM

MIHALY BENCZE - ROMANIA

If $a, b > 1, n \in \mathbb{N}^*$ then:

$$nab + \frac{(ab)^n - 1}{(ab-1)(ab)^n} \geq \frac{a^n - 1}{(a-1)a^{n-1}} + \frac{b^n - 1}{(b-1)b^{n-1}}$$

Solution by Ravi Prakash - New Delhi - India.

$$\textit{For } a, b > 1, m \in \mathbb{N}$$

$$\frac{1}{(ab)^{m+1}} + ab > \frac{1}{a^m} + \frac{1}{b^m}$$

$$\Leftrightarrow 1 + (ab)^{m+2} > ab^{m+1} + a^{m+1}b$$

$$\Leftrightarrow (a^{m+1}b - 1)(ab^{m+1} - 1) > 0$$

$$\textit{which is true}$$

$$\textit{Also, } ab + \frac{1}{ab} > 2$$

$$\textit{Now,}$$

$$\left(ab + \frac{1}{ab}\right) + \left(ab + \frac{1}{a^2b^2}\right) + \left(ab + \frac{1}{a^3b^3}\right) + \ldots + \left(ab + \frac{1}{a^nb^n}\right) >$$

$$> (1+1) + \left(\frac{1}{a} + \frac{1}{b}\right) + \left(\frac{1}{a^2} + \frac{1}{b^2}\right) + \ldots + \left(\frac{1}{a^{n-1}} + \frac{1}{b^{n-1}}\right)$$

$$\Rightarrow nab + \frac{(\frac{1}{ab})(1 - (\frac{1}{ab^n}))}{1 - \frac{1}{ab}}$$

$$> \frac{1 - \frac{1}{a^n}}{1 - \frac{1}{a}} + \frac{1 - \frac{1}{a^n}}{1 - \frac{1}{b}}$$

$$\Rightarrow nab + \frac{(ab)^n - 1}{(ab-1)(ab)^n} > \frac{a^n - 1}{(a-1)a^{n-1}} + \frac{b^n - 1}{(b-1)b^{n-1}}$$

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