PROPOSED PROBLEM

MIHALY BENCZE - ROMANIA

In ΔABC:

$$\sum \frac{-a+b+c}{2a^2-b^2+2c^2-ab+2ac+bc} \geq \frac{9}{10s}$$

Solution by Myagmarsuren Yadamsuren - Darkhan - Mongolia.

$$\sum \frac{(b+c-a)^2}{(b+c-a)\cdot(2a^2-b^2+2c^2-ab+2ac+bc)} \overbrace{\geq}^{Cauchy-Schwarz}$$

$$\geq \frac{(a+b+c)^2}{(b+c-a)\cdot(2a^2-b^2\ldots)+(c+a-b)\cdot(2b^2-c^2+\ldots)+(a+b-c)(2c^2-a^2)} =$$

$$= \frac{(a+b+c)^2}{(3a^2b+3bc^2-b^3+2c^3-2a^3)+(3b^2c+3a^2c-c^3+2a^3-2b^3)+(3ac^2+3ab^2-a^3+2b^3-2c^3)} =$$

$$= \frac{(a+b+c)^2}{3\cdot\sum(a^2b+ab^2)-\sum a^3}$$

$$\frac{(a+b+c)^2}{3\cdot\sum(a^2b+ab^2)-\sum a^3} \geq \frac{9}{10p} \ \textit{(ASSURE)} \Leftrightarrow$$

$$\frac{4p^2}{4\cdot\sum(a^2b+ab^2)-\sum a^2\cdot a} = \frac{4p^2}{4\cdot\sum ab\cdot\sum a-12abc-\sum a^2\cdot\sum a} \geq \frac{9}{10p}$$

$$40p^3 \geq 9(4\cdot\sum ab\cdot\sum a-48pRr-\sum a^2\cdot\sum a)$$

$$40p^3 \geq 9\cdot 2p\cdot(4\sum ab-24Rr-\sum a^2)$$

$$20p^2 \geq 9\cdot(4p^2+16Rr+4r^2-24Rr-2p^2+8Rr+2r^2)$$

$$20p^2 \geq 9\cdot(2p^2+6r^2)$$

$$2p^2 \geq 54r^2$$

$$p^2 \geq 27r^2$$

$$p \geq 3\sqrt{3}r \ \textit{(True)}$$

$\square$

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