$$f:\mathbb{R}\to[a,b],\ a<b$$

**Find:**

$$\Omega=\lim_{n\to\infty}\sum_{k=1}^{n}\frac{(n-k+1)^2 f(k)}{k(1^2+2^2+\cdots+n^2)}$$

*Proposed by Daniel Sitaru – Romania*

*Solution by Saptak Bhattacharya-Kolkata-India*

$$\lim_{n\to\infty}6\sum_{k=1}^{n}\frac{(n-k+1)^2(k)}{kn((n+1)(2n+1)}$$

$$=\lim_{n\to\infty}\frac{6}{n(n+1)(2n+1)}\sum_{k=1}^{n}\frac{(n-k+1)^2 f(k)}{k}$$

***Now,***

$$a\le f(k)\le b,$$

***And,***

$$\lim_{n\to\infty}\frac{6}{n(n+1)(2n+1)}\sum_{k=1}^{n}\frac{(n-k+1)^2 a}{k}$$

$$=\lim_{n\to\infty}\frac{6a}{n(n+1)(2n+1)}\cdot\sum_{k=1}^{n}\frac{n^2+k^2+1-2k-2nk+2n}{k}$$

$$=\lim_{n\to\infty}\frac{6a}{n(n+1)(2n+1)}\cdot\left[(n+1)^2H_n+\frac{n(n+1)}{2}-2n(n+1)\right]$$

$$=\lim_{n\to\infty}\frac{6a}{n(n+1)(2n+1)}\left((n+1)^2H_n-\frac{3n(n+1)}{2}\right)$$

$$=a\lim_{n\to\infty}6\left(\frac{(n+1)H_n}{n(2n+1)}-\frac{3}{2(2n+1)}\right)$$

$$= 6a\left(\frac{1}{2}\lim_{n\to\infty}\frac{H_n}{n}\right)$$

$$= 3a\lim_{n\to\infty}\frac{H_n}{n} = 0$$ ***(Cauchy first theorem)***

***Similarly,***

$$\lim_{n\to\infty}\frac{6b}{n(n+1)(2n+1)}\sum_{k=1}^{n}\frac{(n-k+1)^2}{k} = 0$$

***Thus by squezze theorem, the given limit is*** $0$