SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

Filiala Mehedinți - Mehedinți Branch
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Prove that if $a, b, c \in (0,\infty); abc = 1$ then:

$$be^a + ce^b + ae^c \geq \frac{15}{2}$$

Proposed by Daniel Sitaru – Romania

Solution by Boris Colakovic-Belgrade-Serbia

$$be^a + ce^b + ae^c \overset{AM-GM}{\geq} 3\sqrt[3]{abce^{a+b+c}} = 3\sqrt[3]{e^{a+b+c}}$$

$$\ln\left(be^a + ce^b + ae^c\right) \geq \ln 3 + \frac{1}{3}\ln e^{a+b+c}$$

$$= \ln 3 + \frac{a+b+c}{3} \overset{AM-GM}{\geq} \ln 3 + 1$$

$$be^a + ce^b + ae^c \geq e^{\ln 3 + 1} = e \cdot e^{\ln 3} = 3e > \frac{15}{2}$$