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INEQUALITY IN TRIANGLE 295
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1. In ΔABC

$$\sum(b+c-a)m_a^2 \geq 18pr(R-r).$$

Proposed by Abdikadir Altintas – Afyon – Turkey

Proof.

Using the known identity in triangle $\sum(p-a)m_a^2 = p(p^2-4r^2-7Rr)$

the inequality that we have to prove can be written:

$2p(p^2-4r^2-7Rr) \geq 18pr(R-r) \Leftrightarrow p^2 \geq 16Rr-5r^2$ *(Gerretsen's inequality)*

The equality holds if and only if the triangle is equilateral.

□

Remark

The inequality can be developed:

2. In ΔABC

$$\sum(b+c-na)m_a^2 \geq 9pr\Big[(3-n)R-2r\Big], \text{ where } n \leq 5.$$

Proof.

Using the known identity in triangle $\sum am_a^2 = \frac{1}{2}\cdot p(p^2+5r^2+2Rr)$, *we obtain:*

$$\sum(b+c-na)m_a^2 = \sum\Big[2p-(n+1)a\Big]m_a^2 = 2p\sum m_a^2-(n+1)\sum am_a^2 =$$

$$= 2p\sum m_a^2-(n+1)\cdot\frac{1}{2}\cdot p(p^2+5r^2+2Rr) =$$

$$= 2p\cdot\frac{3}{4}\cdot 2(p^2-r^2-4Rr)-(n+1)\cdot\frac{1}{2}\cdot p(p^2+5r^2+2Rr) =$$

$$= \frac{p}{2}\Big[(5-n)p^2-(5n+11)r^2-(2n+26)Rr\Big].$$

the inequality that we have to prove can be written:

$$\frac{p}{2}\Big[(5-n)p^2-(5n+11)r^2-(2n+26)Rr\Big] \geq 9pr\Big[(3-n)R-2r\Big] \Leftrightarrow$$

$$(5-n)p^2-(5n+11)r^2-(2n+26)Rr \geq 18r\Big[(3-n)R-2r\Big],$$

which follows from Gerretsen's inequality $p^2 \geq 16Rr-5r^2$ *and the condition* $5-n \geq 0$.

The equality holds if and only if the triangle is equilateral.

□

Let's obtain an inequality that have an opposite sense.

3. In ΔABC

$$\sum(b+c-na)m_a^2 \leq p\Big[(10-2n)R^2-(3n+3)Rr+(2-4n)r^2\Big], \text{ where } n \leq 5.$$

Proof.

Using the above proved inequality:

$$\sum(b+c-na)m_a^2 = \frac{p}{2}\Big[(5-n)p^2-(5n+11)r^2-(2n+26)Rr\Big]$$

Gerretsen's inequality $p^2 \leq 4R^2+4Rr+3r^2$ *and the condition* $5-n \geq 0$*, we obtain:*

$$\sum(b+c-na)m_a^2 \leq \frac{p}{2}\Big[(5-n)(4R^2+4Rr+3r^2)-(5n+11)r^2-(2n+26)Rr\Big] =$$

$$= p\Big[(10-2n)R^2-(3n+3)Rr+(2-4n)r^2\Big].$$

The equality holds if and only if the triangle is equilateral.

□

We can write the following double inequality:

4. In ΔABC

$$9pr\Big[(3-n)R-2r\Big] \leq \sum(b+c-na)m_a^2 \leq p\Big[(10-2n)R^2-(3n+3)Rr+(2-4n)r^2\Big], n \leq 5$$

Proposed by Marin Chirciu - Romania

Proof.

*See inequalities **2.** and **3.***

The equality holds if and only if the triangle is equilateral.

□

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