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TRIGONOMETRIC SUBSTITUTIONS IN PROBLEM SOLVING PART 1

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Abstract. In this paper are indicated a few useful trigonometric substitutions for solving problems. Solved problems are also a part of this article.

Case 1: If $x, y, z \in \mathbb{R}; p, q, r \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$1+x^2=1+\tan^2 p; 1+y^2=1+\tan^2 q; 1+z^2=1+\tan^2 r$$

$$F(1+x^2, 1+y^2, 1+z^2)=G\left(\frac{1}{\cos^2 p}; \frac{1}{\cos^2 q}; \frac{1}{\cos^2 r}\right)$$

Case 2: If $x, y, z \in \mathbb{R}; p, q, r \in\left(0, \frac{\pi}{2}\right)$

$$\sqrt{1+x^2}=\frac{1}{\cos p}; \sqrt{1+y^2}=\frac{1}{\cos q}; \sqrt{1+z^2}=\frac{1}{\cos r}$$

$$F(\sqrt{1+x^2}, \sqrt{1+y^2}, \sqrt{1+z^2})=G\left(\frac{1}{\cos p}, \frac{1}{\cos q}, \frac{1}{\cos r}\right)$$

Case 3: If $x, y, z \in \mathbb{R}, m \geq 0, p, q, r \in\left(0, \frac{\pi}{2}\right)$

$$\sqrt{x^2+m^2}=m \tan p; \sqrt{y^2+m^2}=m \tan q; \sqrt{z^2+m^2}=m \tan r$$

$$F(\sqrt{x^2+m^2}, \sqrt{y^2+m^2}, \sqrt{z^2+m^2})=G\left(\frac{m}{\cos p}; \frac{m}{\cos q}; \frac{m}{\cos r}\right)$$

Case 4: If $x, y, z \in \mathbb{R}, p, q, r \in[0, 2\pi]$

$$4x^3-3x=\cos p; 4y^3-4y=\cos q; 4z^3-4z=\cos r$$

$$F(4x^3-3x; 4y^3-3y, 4z^3-3z)=F(\cos p, \cos q, \cos r)$$

Case 5: If $x, y, z \in \mathbb{R}, p, q, r \in[0, 2\pi]$

$$3x-4x^3=\sin p; 3y-4y^3=\sin q; 3z-4z^3=\sin r$$

$$F(3x-4x^3; 3y-4y^3; 3z-4z^3)=F(\sin p, \sin q, \sin r)$$

Case 6: If $x, y, z \in \mathbb{R}; p, q, r \in[0, 2\pi]$

$$2x^2-1=\cos p; 2y^2-1=\cos q; 2z^2-1=\cos r$$

$$F(2x^2-1; 2y^2-1; 2z^2-1)=F(\cos p, \cos q, \cos r)$$

Case 7: If $x, y, z \in \mathbb{R}, p, q, r \in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\frac{2x}{1-x^2}=\tan p; \frac{2y}{1-y^2}=\tan q; \frac{2z}{1-z^2}=\tan r$$

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$$F\Big(\frac{2x}{1-x^2};\frac{2y}{1-y^2};\frac{2z}{1-z^2}\Big)=G(\tan p,\tan q,\tan r)$$

Case 8: If $x,y,z\in\mathbb{R};p,q,r\in\Big(-\frac{\pi}{2},\frac{\pi}{2}\Big)$

$$\frac{2x}{1+x^2}=\tan p;\frac{2y}{1+y^2}=\tan q;\frac{2z}{1+z^2}=\tan r$$

$$F\Big(\frac{2x}{1+x^2};\frac{2y}{1+y^2};\frac{2z}{1+z^2}\Big)=G(\sin 2p,\sin 2q,\sin 2r)$$

Case 9: If $x,y,z\in\mathbb{R};p,q,r\in\Big[0,\frac{\pi}{2}\Big)\cup\Big[\pi,\frac{3\pi}{2}\Big)$

$$x=\frac{1}{\cos p};y=\frac{1}{\cos q};z=\frac{1}{\cos r}$$

$$F(x^2-1;y^2-1;z^2-1)=G(\tan^2 p,\tan^2 q,\tan^2 r)$$

Case 10: If $x,y,z\in\mathbb{R};|x|,|y|,|z|\geq 1,p,q,r\in\Big[0,\frac{\pi}{2}\Big)$

$$F(\sqrt{x^2-1};\sqrt{y^2-1};\sqrt{z^2-1})=G(\tan p,\tan q,\tan r)$$

$$x=\frac{1}{\cos p};y=\frac{1}{\cos q};z=\frac{1}{\cos r}$$

Case 11: If $x,y,z\in\mathbb{R};|x|,|y|,|z|\geq m,p,q,r\in\Big[0,\frac{\pi}{2}\Big)$

$$x=\frac{m}{\cos p};y=\frac{m}{\cos q};z=\frac{m}{\cos r}$$

$$F(\sqrt{x^2-m^2},\sqrt{y^2-m^2},\sqrt{z^2-m^2})=G(m\tan p,m\tan q,m\tan r)$$

Case 12: If $x,y,z\in\mathbb{R};xy\neq 1;yz\neq 1;zx\neq 1,p,q,r\in\mathbb{R}$

$$x=\tan p;y=\tan q;z=\tan r$$

$$F\Big(\frac{x+y}{1-xy};\frac{y+z}{1-yz};\frac{z+x}{1-zx}\Big)=G\Big(\tan(p+q),\tan(q+r),\tan(r+p)\Big)$$

Problem 1. If $x\in\mathbb{R};|x|\leq 1;n\in\mathbb{N}$ then:

$$(1-x)^n+(1+x)^n\leq 2^n$$

Proof.

$$|x|\leq 1\Rightarrow(\exists)t\in\Big[0,\frac{\pi}{2}\Big],x=\cos 2t$$

Remains to prove:

$$(1-\cos 2t)^n+(1+\cos 2t)^n\leq 2^n$$
$$(2\sin^2 t)^n+(2\cos^2 t)^n\leq 2^n$$
$$\sin^{2n}t+\cos^{2n}t\leq 1$$

which is obviously because:

$$\left.\begin{array}{l}\sin^{2n}t\leq\sin^2 t\\ \cos^{2n}t\leq\cos^2 t\end{array}\right\}\Rightarrow\sin^{2n}t+\cos^{2n}t\leq\sin^2 t+\cos^2 t=1$$

□

Problem 2 If $a\in(-\infty,-1)\cup(1,\infty)$ then:

$$\sqrt{a^2-1}+\sqrt{3}\leq 2|a|$$

Proof.

$$|a|>1 \Rightarrow (\exists)|a|=\frac{1}{\cos\alpha};\alpha\in\Big[0,\frac{\pi}{2}\Big)$$

Remains to prove:

$$\sqrt{\frac{1}{\cos^2\alpha}-1}+\sqrt{3}\leq\frac{2}{\cos\alpha}\Leftrightarrow\tan\alpha+\sqrt{3}\leq\frac{2}{\cos\alpha}$$
$$\frac{1}{2}\sin\alpha+\sqrt{3}\cos\alpha\leq 1\Leftrightarrow\sin\Big(\alpha+\frac{\pi}{3}\Big)\leq 1$$

□

Problem 3

If $a\in(0,1)$ then:

$$|4(a^3-\sqrt{(1-a^2)^3})-3(a-\sqrt{1-a^2})|\leq\sqrt{2}$$

Proof.

$$|a|\leq 1\Rightarrow(\exists)x\in\Big[0,\frac{\pi}{2}\Big);a=\cos x$$

The inequality can be written:

$$|4(\cos^3 x-\sqrt{(1-\cos^2 x)^3})-3(\cos x-\sqrt{1-\cos^2 x})|\leq\sqrt{2}$$
$$|4(\cos^3 x-\sin^3 x)-3(\cos x-\sin x)|\leq\sqrt{2}$$
$$|(4\cos^3 x-3\cos x)+(3\sin x-4\sin^3 x)|\leq\sqrt{2}$$
$$|\cos 3x+\sin 3x|\leq\sqrt{2}$$
$$\Big|\cos 3x\cdot\frac{\sqrt{2}}{2}+\sin 3x\cdot\frac{\sqrt{2}}{2}\Big|\leq 1$$
$$\Big|\sin\Big(3x+\frac{\pi}{4}\Big)\Big|\leq 1$$

□

Problem 4

If $x,y,z\in\mathbb{R}$ then:

$$\left|\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}\right|\leq\frac{1}{2}$$

Proof.

$$x,y\in\mathbb{R}\Rightarrow(\exists)p,q\in\Big(-\frac{\pi}{2},\frac{\pi}{2}\Big);x=\tan p;y=\tan q$$
$$\frac{(x+y)(1-xy)}{(1+x^2)(1+y^2)}=\frac{(\tan p+\tan q)(1-\tan p\tan q)}{(1+\tan^2 p)(1+\tan^2 q)}=$$
$$=\frac{\sin(p+q)\cos(p+q)\cos^2 p\cos^2 q}{\cos p\cos q\sin p\sin q}=\sin(p+q)\cos(p+q)=$$
$$=\frac{1}{2}\sin 2(p+q)$$

Remains to prove that:

$$\Big|\frac{1}{2}\sin 2(p+q)\Big|\leq\frac{1}{2}\Leftrightarrow|\sin 2(p+q)|\leq 1$$

□

Problem 5

If $a, b \in \mathbb{R}$ then:

$$a^2(1+b^4)+b^2(1+a^4) \leq (1+a^4)(1+b^4)$$

Proof.

$$a^2 \geq 0, b^2 \geq 0 \Rightarrow (\exists)p, q \in \Big[0, \frac{\pi}{2}\Big), a^2 = \tan p, b^2 = \tan q$$

$$\tan p(1+\tan^2 q) + \tan q(1+\tan^2 p) \leq (1+\tan^2 p)(1+\tan^2 q)$$

$$\tan p \cdot \frac{1}{\cos^2 q} + \tan q \cdot \frac{1}{\cos^2 p} \leq \frac{1}{\cos^2 p} \cdot \frac{1}{\cos^2 q}$$

$$\tan p \cos^2 p + \tan q \cos^2 q \leq 1$$

$$\sin p \cos p + \sin q \cos q \leq 1$$

$$2\sin p \cos p + 2\sin q \cos q \leq 2$$

$$\sin 2p + \sin 2q \leq 2$$

which its obvious because $\sin 2p \leq 1; \sin 2q \leq 1$ □

Problem 6

If $x, y \geq 0; x+y=1$ then:

$$\Big(x^2 + \frac{1}{x^2}\Big) + \Big(y^2 + \frac{1}{y^2}\Big) \geq \frac{17}{2}$$

Proof.

$$x, y \geq 0 \Rightarrow (\exists)p \in [0, 2\pi); x = \sin^2 p; y = \cos^2 p$$

$$\sin^4 p + \frac{1}{\cos^2 p} + \cos^4 p + \frac{1}{\sin^4 p} =$$

$$= (\sin^4 p + \cos^4 p)\Big(1 + \frac{1}{\sin^4 p \cos^4 p}\Big) =$$

$$= (1 - 2\sin^2 p \cos^2 p)\Big(1 + \frac{16}{\sin^4 2p}\Big) =$$

$$= (1 - \frac{\sin^2 2p}{2}\Big)\Big(1 + \frac{16}{\sin^4 2p}\Big) \geq$$

$$\geq \Big(1 - \frac{1}{2}\Big)\Big(1 + \frac{16}{1}\Big) = \frac{1}{2} \cdot 17 = \frac{17}{2}$$

□

Problem 7

If $a, b \in \mathbb{R}$ then:

$$a^2 + (a-b)^2 \geq \frac{3-\sqrt{5}}{2}(a^2+b^2)$$

Proof. If $b = 0$ inequality ca be written:

$$2a^2 \geq \frac{3-\sqrt{5}}{2}a^2 \Leftrightarrow a^2(1+\sqrt{5}) \geq 0$$

If $b \neq 0$, dividing by b^2:

$$\frac{a^2}{b^2} + \Big(\frac{a}{b} - 1\Big)^2 \geq \frac{3-\sqrt{5}}{2}\Big(\frac{a^2}{b^2} + 1\Big)$$

$$\frac{a}{b} \in \mathbb{R} \Rightarrow (\exists)p \in \Big(-\frac{\pi}{2}, \frac{\pi}{2}\Big); \frac{a}{b} = \tan p$$

$$\tan^2 p + (\tan p - 1)^2 \geq \frac{3-\sqrt{5}}{2}(\tan^2 p + 1)$$

$$\sin^2 p + (\sin p - \cos p)^2 \geq \frac{3-\sqrt{5}}{2}$$

$$1 + \sin^2 p - 2\sin p \cos p \geq \frac{3-\sqrt{5}}{2}$$

$$2 + 2\sin^2 p - 2\sin 2p \geq 3 - \sqrt{5}$$

$$1 - 2\sin^2 p + 2\sin 2p \leq \sqrt{5}$$

$$\cos 2p + 2\sin 2p \leq \sqrt{5}$$

$$\frac{1}{\sqrt{5}}\cos 2p + \frac{2}{\sqrt{5}}\sin 2p \leq 1$$

$$\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{2}{\sqrt{5}}\right)^2 = 1 \Rightarrow (\exists) q \in \left(0, \frac{\pi}{2}\right);$$

$$\frac{1}{\sqrt{5}} = \sin q; \frac{2}{\sqrt{5}} = \cos q$$

$$\sin q \cos 2p + \cos q \sin 2p \leq 1$$

$$\sin(2p + q) \leq 1$$

□

Problem 8
If $a, b \in [0,1]$ then:

$$|a\sqrt{1-b^2} + b\sqrt{1-a^2} + \sqrt{3}(ab - \sqrt{(1-a^2)(1-b^2)})| \leq 2$$

Proof.

$$a, b \in [0,1] \Rightarrow (\exists) p, q \in \left[0, \frac{\pi}{2}\right], a = \sin p, b = \sin q$$

$$\left| \sin p\sqrt{1-\sin^2 q} + \sin q\sqrt{1-\sin^2 p} + \sqrt{3}\left(\sin p \sin q - \sqrt{(1-\sin^2 p)(1-\sin^2 q)}\right) \right| \leq 2$$

$$\left| \sin p \cos q + \sin q \cos p + \sqrt{3}(\sin p \sin q - \cos p \cos q) \right| \leq 2$$

$$\left| \sin(p+q) - \sqrt{3}\cos(p+q) \right| \leq 2$$

$$\left| \frac{1}{2}\sin(p+q) - \frac{\sqrt{3}}{2}\cos(p+q) \right| \leq 1$$

$$\left| \sin\left(p + q - \frac{\pi}{3}\right) \right| \leq 1$$

□

Problem 9:
Solve the following equation:

$$x^3 - 3x + a(1 - 3x^2) = 0; a \in \mathbb{R}$$

Proof.

$$x \in \mathbb{R} \Rightarrow \exists b \in \Big(-\frac{\pi}{2}, \frac{\pi}{2}\Big); x = \tan b$$

$$a(1 - 3\tan^2 b) = 3\tan b - \tan^3 b \Rightarrow a = \frac{3\tan b - \tan^3 b}{1 - 3\tan^2 b}$$

$$a = \tan 3b \Rightarrow 3b = \arctan a + k\pi; k \in \mathbb{Z} \Rightarrow b = \frac{1}{3}\arctan a + \frac{k\pi}{3}; k \in \mathbb{Z}$$

$$x = \tan b = \tan\Big(\frac{1}{3}\arctan a + \frac{k\pi}{3}\Big); k \in \{-2, 0, 2\}$$

If $a = 1$ the equation: $x^3 - 3x^2 - 3x + 1 = 0$ has the solutions:

$$x = \tan\Big(\frac{\pi}{12} + \frac{k\pi}{3}\Big); k \in \{-2, 0, 2\}, x_1 = \tan\frac{\pi}{12}; x_2 = \tan\frac{\pi}{4}; x_3 = \tan\Big(-\frac{7\pi}{4}\Big)$$

If $a = 2$ the equation: $x^3 - 6x^2 - 3x + 2 = 0$ has the solutions:

$$x = \tan\Big(\frac{1}{3}\arctan 2 + \frac{k\pi}{3}\Big)$$

$$k \in \{-2, 0, 2\}, x_1 = \tan\Big(\frac{1}{3}\arctan 2\Big); x_2 = \tan\Big(\frac{1}{3}\arctan 2 + \frac{2\pi}{3}\Big); x_3 = \tan\Big(\frac{1}{3}\arctan 2 - \frac{2\pi}{3}\Big)$$

□

Problem 10:

Solve the following equation:

$$4x^3 - 4x + a(x^4 - 6x^2 + 1) = 0; a \in \mathbb{R}$$

Proof.

$$x \in \mathbb{R} \Rightarrow \exists b \in \Big(-\frac{\pi}{2}, \frac{\pi}{2}\Big); x = \tan b; a(\tan^4 b - 6\tan^2 b + 1) = 4\tan b - 4\tan^3 b$$

$$a = \frac{4\tan b - 4\tan^3 b}{\tan^4 b - 6\tan^2 + 1} \Rightarrow a = \tan 4b \Rightarrow b = \frac{1}{4}\arctan a; x = \tan\Big(\frac{1}{4}\arctan a + \frac{k\pi}{4}\Big); k \in \mathbb{Z}$$

If $a = 1$ the equation $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ has the solutions:

$$x_1 = \tan\frac{\pi}{16}; x_2 = \tan\frac{5\pi}{16}; x_3 = \tan\Big(-\frac{3\pi}{16}\Big); x_4 = \tan\frac{9\pi}{16}$$

□

Problem 11.

Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in [-1, 1], x_{n+1} = 2x_n^2 - 1; n \geq 1$$

Proof.

$$x_1 = a \in [-1, 1] \Rightarrow (\exists) b \in [0, 2\pi]; a = \cos b; b = \arccos a$$

$$x_2 = 2x_1^2 - 1 = 2\cos^2 b - 1 = \cos(2b); x_3 = 2x_2^2 - 1 = 2\cos^2(2b) - 1 = \cos(2^2 b)$$

$$x_4 = 2x_3^2 - 1 = 2\cos^2(2^2 b) - 1 = \cos(2^3 b).$$

Through induction we can prove that:

$$x_n = \cos(2^{n-1} b) = \cos(2^{n-1}\arccos a)$$

□

Problem 12.
Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in [-1,1]; x_{n+1} = 1 - 2x_n^2; n \geq 1$$

$$x_2 = 1 - 2x_1^2 = 1 - 2\cos^2 b = \cos(2b); x_3 = 1 - 2x_2^2 = 1 - 2\cos^2(2b) = \cos(2^2 b)$$

$$x_4 = 1 - 2x_3^2 = 1 - 2\cos^2(2^2 b) = \cos(2^3 b)$$

Through induction we can prove that:

$$x_n = \cos(2^{n-1} b) = \cos(2^{n-1} \arcsin a)$$

Problem 13.
Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in [-1,1]; x_{n+1} = x_n(3 - 4x_n^2); n \geq 1$$

Proof.

$$x_1 = a \in [-1,1] \Rightarrow (\exists) b \in [0, 2\pi]; a = \sin b; b = \arcsin a$$

$$x_2 = x_1(3 - 4x_1^2) = \sin b(3 - 4\sin^2 b) = \sin(3b)$$

$$x_3 = x_2(3 - 4x_2^2) = \sin 3b(3 - 4\sin^2 3b) = \sin(3^2 b)$$

$$x_4 = x_3(3 - 4x_3^2) = \sin(3^2 b)\Big(2 - 4\sin^4(3^2 b)\Big) = \sin(3^3 b)$$

Through induction we can prove that:

$$x_n = \cos(3^{n-1} b) = \cos(3^{n-1} \arccos a)$$

□

Problem 14
Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in [-1,1]; x_{n+1} = x_n(4x_n^2 - 3); n \geq 1$$

Proof.

$$x_1 = a; a \in [-1,1] \Rightarrow (\exists) b \in [0, 2\pi]; a = \cos b; b = \arccos a$$

$$x_2 = x_1(4x_1^2 - 3) = \cos b(4\cos^2 b - 3) = \cos(3b)$$

$$x_3 = x_2(4x_2^2 - 3) = \cos 3b(4\cos^2 3b - 3) = \cos(3^2 b)$$

Through induction we can prove that:

$$x_n = \cos(3^{n-1} b) = \cos(3^{n-1} \arccos a)$$

□

Problem 15.
Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in \mathbb{R}; x_{n+1} = \frac{2x_n}{1 - x_n^2}; n \geq 1$$

Proof.

$$x_1 = a \in [-1,1] \Rightarrow (\exists) b \in \Big(-\frac{\pi}{2}, \frac{\pi}{2}\Big); a = \tan b; b = \arctan a$$

$$x_2 = \frac{2x_1}{1 - x_1^2} = \frac{2\tan b}{1 - \tan^2 b} = \tan(2b); x_3 = \frac{2x_2}{1 - x_2^2} = \frac{2\tan(2b)}{1 - \tan^2(2b)} = \tan(2^2 b)$$

Through induction we can prove that:

$$x_n = \tan(2^{n-1} b) = \tan(2^{n-1} \arctan a)$$

□

Problem 16.
Find the formula of the general term of the sequence given by the relationships:

$$x_1 = a; a \in \mathbb{R}; x_{n+1} = \frac{3x_n - x_n^3}{1 - 3x_n^2}; n \geq 1$$

Proof.

$$x_1 = a \in [-1, 1] \Rightarrow (\exists) b \in \left(-\frac{\pi}{2}, \frac{\pi}{2} \right); a = \tan b; b = \arctan a$$

$$x_2 = \frac{3x_1 - x_1^3}{1 - 3x_1^2} = \frac{3\tan b - \tan^3 b}{1 - 3\tan^2 b} = \tan(3b); x_3 = \frac{3x_2 - x_2^3}{1 - 3x_2^2} = \frac{3\tan(3b) - \tan^3(3b)}{1 - 3\tan^2(3b)} = \tan(3^2 b)$$

Through induction we prove that:

$$x_n = \tan(3^{n-1} b) = \tan(3^{n-1} \arctan a)$$

□

Proposed Problems:

17. If $x \geq 1$ then:

$$\sqrt{x-1} + \sqrt{x(\sqrt{x-1})} < x$$

$$\left(\text{Use: } x = \frac{1}{\cos^2 p}; p \in \left[0, \frac{\pi}{2}\right) \right)$$

18. If $|a| \leq 1$ then:

$$\sqrt{1 + \sqrt{1 - a^2}} \left[\sqrt{(1+a)^3} - \sqrt{(1-a)^3} \right] \leq 2\sqrt{2} + \sqrt{2 - 2a^2}$$

$$\left(\text{Use: } a = \cos p; p \in [0, \pi] \right)$$

19. If $|x| \leq 1$ then:

$$\frac{\sqrt{3} - 2}{2} \leq \sqrt{3}x^2 + x\sqrt{1 - x^2} \leq \frac{\sqrt{3} + 2}{2}$$

$$\left(\text{Use: } x = \cos p; p \in [0, \pi] \right)$$

20. If $a \in [0, 2]$ then:

$$|\sqrt{2a - a^2} - \sqrt{3}a + \sqrt{3}| \leq 2$$

$$\left(\text{Use: } a - 1 = \cos p; p \in [0, \pi] \right)$$

21. If $|a| \geq 1$ then:

$$\left| \frac{\sqrt{a^2 - 1} + \sqrt{3}}{a} \right| \leq 2$$

$$\left(\text{Use: } a = \frac{1}{\cos p}; p \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right) \right)$$

22. If $|a| \geq 1$ then:

$$-4 \leq \frac{5 - 12\sqrt{a^2 - 1}}{a^2} \leq 9$$

$$\left(\text{Use: } a = \frac{1}{\cos p}; p \in \left[0, \frac{\pi}{2}\right) \cup \left[\pi, \frac{3\pi}{2}\right) \right)$$

TO BE CONTINUED!

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