$d_D = d(M,(ABC))$. Prove that: $\frac{m_A}{xh_A+yd_A}+\frac{m_B}{xh_B+yd_B}+\frac{m_C}{xh_C+yd_C}+\frac{m_D}{xh_D+yd_D}\ge\frac{16}{4x+y}$

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase-Romania

S.2206 Let $x,y>0, g_a, g_b, g_c$ −Gergonne's cevians and $M\in Int(\Delta ABC)$.

If $d_a = d(M,BC), d_b = d(M,CA)$ and $d_c = d(M,AB)$, then:

$$\frac{g_a}{xh_a+yd_a}+\frac{g_b}{xh_b+yd_b}+\frac{g_c}{xh_c+yd_c}\ge\frac{9}{3x+y}$$

Proposed by D.M. Bătinețu-Giurgiu, Mihaly Bencze-Romania

S.2207 If $t>0$, then in ΔABC holds:

$$(r_a^2+t)(r_b^2+t)(r_c^2+t)\ge\frac{27\sqrt{3}}{4}\cdot t^2F$$

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase-Romania

S.2208 If $a,b,c>0$, then: $(a^2+1936)(b^2+2025)(c^2+2016)\ge\frac{3}{4}(2070a+2024b+1980c)^2$

Proposed by D.M. Bătinețu-Giurgiu-Romania

S.2209 If $t\ge 0, u,v>0$ and $X,Y\in Int(\Delta ABC)$, $x_a = d(X,BC), y_a = d(Y,BC)$ and analogous x_b, x_b, y_b, y_c, then:

$$\frac{a^{t+2}}{(ux_a+vy_a)^t}+\frac{b^{t+2}}{(ux_b+vy_b)^t}+\frac{c^{t+2}}{(ux_c+vy_c)^t}\ge\frac{2^{t+2}\left(\sqrt{3}\right)^{t+1}}{(u+v)^t}\cdot F$$

Proposed by D.M. Bătinețu-Giurgiu, Claudia Nănuți-Romania

S.2210 In $\Delta ABC, g_a, g_b, g_c$ −Gergonne's cevians, holds:

$$\frac{g_b+g_c}{h_a^3}+\frac{g_c+g_a}{h_b^3}+\frac{g_a+g_b}{h_c^3}\ge\frac{2\sqrt{3}}{F}$$

Proposed by D.M. Bătinețu-Giurgiu, Mihaly Bencze-Romania

S.2211 Let $(a_n)_{n\ge1}$ an sequence of real numbers strictly positive such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n\cdot a_n}=a>0$ and $\gamma_n=-\log n+\sum_{k=1}^{n}\frac{1}{k}, \gamma$ −Euler constant. Find:

$$\Omega=\lim_{n\to\infty}(\gamma_{n+1}\gamma_n-\gamma^2)\sqrt[n]{a_n}$$

Proposed by D.M. Bătinețu-Giurgiu, Daniel Sitaru-Romania

S.2212 Let $(a_n)_{n\ge1}, (b_n)_{n\ge1}, (c_n)_{n\ge1}$ sequences of real numbers strictly positive such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n^3\cdot a_n}=a>0$, $\lim\limits_{n\to\infty}\frac{b_{n+1}}{n^2\cdot b_n}=b>0$, $\lim\limits_{n\to\infty}c_n=c>0$ and

$\lim_{n\to\infty} n(c_n - c) = d > 0$. Find: $\Omega = \lim_{n\to\infty} \sqrt[n]{\frac{a_n}{b_n}}(c_{n+1}c_n - c^2)$.

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase-Romania

S.2213 In ΔABC the following relationship holds:

$$\frac{3}{4(2R^2+r^2)} \leq \frac{1}{a^2+2b^2} + \frac{1}{b^2+2c^2} + \frac{1}{c^2+2a^2} \leq \frac{1}{12r^2}$$

Proposed by Nguyen Van Canh-Vietnam

S.2214 Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that $f(x+y) = f(x)f(x-2y) + 2021xy, \forall x, y \in \mathbb{R}$

Proposed by Nguyen Van Canh-Vietnam

S.2215 In ΔABC the following relationship holds:

$$a) \sum_{cyc} m_a^2 + r(R-2r) \leq \sum_{cyc} r_a^2,\ \ b) \sum_{cyc} r_a^2 \leq \sum_{cyc} m_a^2 + 16(R^2-4r^2)$$

$$c) \min\{h_a, h_b, h_c\} \leq \frac{\sum h_a}{3} \leq \frac{\sum h_a + r(R-2r)}{3} \leq \frac{\sum m_a}{3}$$

Proposed by Nguyen Van Canh-Vietnam

S.2216 If $a, b, c > 0$ and $0 < \lambda \leq 2$ then:

$$\sum_{cyc} \frac{b+c}{a} \geq \lambda \sqrt{\sum_{cyc} ab} \cdot \sqrt{\sum_{cyc} \frac{1}{a^2}}$$

Proposed by Nguyen Van Canh-Vietnam

S.2217 Let $a, b > 0, c \geq 0$. Prove that:

$$\frac{1}{b^2+bc+c^2} + \frac{1}{c^2+ca+a^2} \geq \frac{2(a+b)}{ab(a+b)+bc(b+c)+ca(c+a)} \geq \frac{2(a+b)}{a^3+b^3+c^3+abc}$$

Proposed by Nguyen Van Canh-Vietnam

S.2218 Let $x, y, z, t > 0, x+y+z+t = 4$. Prove that:

$$3\sum_{cyc} x^2 \geq 16 - 4 \cdot \sqrt[3]{xyzt}$$

Proposed by Nguyen Van Canh-Vietnam

S.2219 Let $a, b, c \in \mathbb{R}$ such that: $|ax^4 + bx^2 + c|\sqrt{1-x^2} \leq 1, \forall |x| \leq 1$. Prove that $|a| \leq 16$.

Proposed by Nguyen Van Canh-Vietnam

S.2220 Find all numbers $\alpha > 0$ such that:

$$\frac{a^4+b^4+c^4}{ab+bc+ca}+\frac{3abc}{a+b+c}\geq\alpha(a^2+b^2+c^2);\forall a,b,c>0$$

Proposed by Nguyen Van Canh-Vietnam

S.2221 Find all polynomials $p:\mathbb{R}\to\mathbb{R}$ such that:

$$p(x)+2p(x^2)+2p(x^3)=5x^6+4x^4-6x^2+x-1,\forall x\in\mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

S.2222 In ΔABC, g_a −Gergonne's cevian, the following relationship holds:

$$a)\ \sum_{cyc}g_a^2+\frac{r^2(R-2r)}{3R}\leq\min\left\{\sum_{cyc}w_a^2,\sum_{cyc}m_a^2,1\right\}$$

$$b)\ \sum_{cyc}g_a^2+\frac{r^2(R-2r)}{2R}\leq\max\left\{\sum_{cyc}w_a^2,\sum_{cyc}m_a^2,s^2\right\}$$

Proposed by Nguyen Van Canh-Vietnam

S.2223 Let $\alpha,\beta>0$. Find all polynomials $p:\mathbb{R}\to\mathbb{R}$ such that: $\alpha p(\beta x^{2021})+\frac{\beta}{\alpha}x^{\frac{\beta}{\alpha}}=\beta p(\alpha x^{2021})$

Proposed by Nguyen Van Canh-Vietnam

S.2224 Let $a,b,c>0, ab+bc+ca\leq a+b+c$ then:

$$\sum_{cyc}\sqrt{\frac{a}{b}}\geq\frac{9(ab+bc+ca)}{\left(\sqrt{a}+\sqrt{b}+\sqrt{c}\right)^2}$$

Proposed by Nguyen Van Canh-Vietnam

S.2225 Solve in $\mathbb{R}$: $a)$ $\tan x-\cot x=|\sin x-\cos x|$, $b)$ $\log x-\lg x=|e^x-1|$, $c)$ $e^{\sqrt{x}}\geq x$

Proposed by Nguyen Van Canh-Vietnam

S.2226 In ΔABC, ω −Brocard's angle, holds:

$$\frac{1}{2}\sum_{cyc}\left(\frac{h_b}{h_c}+\frac{h_c}{h_b}\right)\frac{w_a^2}{h_a^2}\leq 1+\frac{1}{2\sin^2\omega}$$

Proposed by Bogdan Fuștei-Romania

S.2227 In ΔABC, $(r_i)_{i=1,2,3}$ −Malfatti's radies, holds:

$$\frac{r_1}{r_a}+\frac{r_2}{r_b}+\frac{r_3}{r_c}\leq 1-\frac{3\left(3-\sqrt{3}\right)}{2}\cdot\frac{r}{s}$$

Proposed by Bogdan Fuștei-Romania

S.2228 In $\Delta ABC, (r_i)_{i=1,2,3}$ −Malfatti's radies, holds:

$$2\sum_{cyc}\sqrt{r_1 r_2}+s\leq m_a+m_b+m_c$$

Proposed by Bogdan Fuștei-Romania

S.2229 If $X\in INt(\Delta ABC), XA'=XB'=XC'=R, R$ −circumradii, $XA'\perp BC, XB'\perp CA, XC'\perp AB, a', b', c', I_{a'}, I_{b'}, I_{c'}$ sides and excenters in $\Delta A'B'C'$ then:

$$[I_{a'}I_{b'}I_{c'}]=2R^2\left(\cos\frac{A}{2}+\cos\frac{B}{2}+\cos\frac{C}{2}\right)$$

Proposed by Mehmet Șahin-Turkiye

S.2230 In $\Delta ABC, n_a$ −Nagel's cevian, holds:

$$n_a n_b n_c\sum_{cyc}\left(\frac{n_a h_b}{n_b h_a}+\frac{n_b h_a}{n_a h_b}\right)\geq s^3$$

Proposed by Bogdan Fuștei-Romania

S.2231 If $a, b, c>0$ then:

$$\frac{a}{3a^2+2b^2+c^2}\geq\frac{1}{18}\left(\frac{2}{b}+\frac{1}{c}\right)$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

S.2232 In $\Delta ABC, P\in(AB), Q\in(BC), R\in(CA)$ such that $AP=a, PB=b, BQ=c, QC=d,$

$CR=e, RA=f$, then prove:

$$\frac{bc}{(a+b)(c+d)}+\frac{de}{(d+c)(e+f)}+\frac{af}{(a+b)(e+f)}<1$$

Proposed by Neculai Stanciu-Romania

S.2233 If $A=\begin{pmatrix} 1 & i & -1 & -i & \dots & i \\ i & 1 & i & -1 & \dots & 1 \\ 1 & i & 1 & i & \dots & -i \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ i & -1 & -i & 1 & \dots & 1 \end{pmatrix}\in M_{2022}(\mathbb{C})$, then compute $\det A$.

Proposed by Neculai Stanciu-Romania

S.2234 If $a>0$ and $E(x)=\frac{a^x}{a^x+\sqrt{a}}$, then compute: $E\left(\frac{1}{2021}\right)+E\left(\frac{2}{2021}\right)+\cdots+E\left(\frac{2020}{2021}\right)$

Proposed by Neculai Stanciu-Romania

S.2235 If $x, y, z\geq 1, x+y+z=6$, then prove:

$$\frac{x^2+3}{3x^2+1}+\frac{y^2+3}{3y^2+1}+\frac{z^2+3}{3z^2+1}\ge\frac{21}{13}$$

Proposed by Neculai Stanciu-Romania

S.2236 If $f: D\subset\mathbb{R}\to\mathbb{R}, f(x)=\frac{1+4x}{3+2x}$, then find $f_n(x)=\left(\underbrace{f\circ f\circ\dots\circ f}_{n-\text{times } f}\right)(x), \forall n\in\mathbb{N}^*$.

Proposed by Neculai Stanciu-Romania

S.2237 If $a,b,c>0, a^4+b^4+c^4=3$, then prove that:

$$2(a^2+b^2)(b^2+c^2)(c^2+a^2)\ge(a^2b^2+b^2c^2+c^2a^2+abc)^2$$

Proposed by Neculai Stanciu, Marius Drăgan-Romania

S.2238 If $m,n\ge 2$ then prove that in any convex quadrilateral $ABCD$ with usual notations is true the inequality:

$$\frac{b^n+c^n+d^n}{a^m}+\frac{c^n+d^n+a^n}{b^m}+\frac{d^n+a^n+b^n}{c^m}+\frac{a^n+b^n+c^n}{d^m}\ge 12\left(\frac{2}{s}\right)^{m+n}$$

Proposed by Marius Drăgan, Neculai Stanciu-Romania

S.2239 In ΔABC, $[ABC]$ –area and $P\in(AB), Q\in(BC), R\in(CA)$ such that $AP=a, PB=b$, $BQ=c, QC=d, CR=e, RA=f$, then determine $\frac{[PQR]}{[ABC]}$ in terms of a,b,c,d,e,f.

Proposed by George Florin Șerban, Neculai Stanciu-Romania

S.2240 Solve in $\mathbb{Z}\times\mathbb{Z}$ the following equation: $a^2+b^2-2a+b=5$

Proposed by Neculai Stanciu-Romania

S.2241 Solve in $\mathbb{R}^*$ the equation $\left[\frac{1}{x}\right]+\left[\frac{3}{x}\right]=4$, where $[*]$ –GIF.

Proposed by Neculai Stanciu-Romania

S.2242 If $m\in[0,\infty), x,y,z,t\in(0,\infty)$, then in any triangle ABC holds:

$$\sum_{cyc}\frac{\left(xm_a^2+ym_b^2\right)^{m+1}}{(zw_c^2+th_a^2)^m}\ge\frac{(x+y)^{m+1}}{(z+t)^m}3\sqrt{3}F$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

S.2243 If $f:(0,\infty)\to\mathbb{R}, f(x)=\begin{cases}\frac{\{x\}+1}{x}, \{x\}\le\frac{1}{3+n}\\ \frac{1-\{x\}}{x}, \{x\}>\frac{1}{3+n}\end{cases}$, $\{x\}$ – is fractional part of x and $n\in\mathbb{N}$, then find:

$$\Omega = \lim_{n\to\infty}\int_{\frac{1}{5+n}}^{\frac{1}{2+n}} f(x)\,dx$$

Proposed by Neculai Stanciu-Romania

S.2244 If $ABCDA'B'C'D'$ is a cube with O the center of face $ABCD$ and O' the center of face $BCC'B'$, then compute the distance between the lines $B'O$ and BO' in terms of side of cube.

Proposed by Neculai Stanciu-Romania

S.2245 If $e_n = \left(1+\frac{1}{n}\right)^n$ and $\gamma = -\log n + \sum_{k=1}^{n}\frac{1}{k}$, then compute:

$$\Omega = \lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{(2n+1)!!\,\gamma_n}} - \frac{n^2}{\sqrt[n]{(2n-1)!!\,e_n}}\right)$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

S.2246 Solve in $M_2(\mathbb{R})$ the following equation $A^n = \begin{pmatrix}4 & 8\\ 3 & 6\end{pmatrix}, n \in \mathbb{N}^*$.

Proposed by Neculai Stanciu-Romania

S.2247 If $a, m \in \mathbb{R}_+$ and $b, c, d, x, y, z \in \mathbb{R}_+^*, X = x + y + z, cX > d\max\{x, y, z\}$, then

$$\left(\frac{aX+bx}{cX-dx}\right)^{m+1} + \left(\frac{aX+by}{cX-dy}\right)^{m+1} + \left(\frac{aX+bz}{cX-dz}\right)^{m+1} \ge \frac{3(3a+b)^{m+1}}{(3c-d)^{m+1}}$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

S.2248 If $m, n \ge 1$ then prove that in any bicentric quadrilateral $ABCD$ with usual notations is true the inequality:

$$\frac{b^n+c^n+d^n}{a^m} + \frac{c^n+d^n+a^n}{b^m} + \frac{d^n+a^n+b^n}{c^m} + \frac{a^n+b^n+c^n}{d^m} \ge \frac{12}{2^{\frac{m+n}{2}}R^{m-n}}$$

Proposed by Marius Drăgan, Neculai Stanciu-Romania

S.2249 In which triangle ABC holds the identity:

$$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{a\tan A + b\tan B}{a+b}?$$

Proposed by Neculai Stanciu-Romania

S.2250 If $x, y, z > 0, xyz \ge 7 + 5\sqrt{2}$ then prove that: $x^2 + y^2 + z^2 - 2(x + y + z) \ge 3$

Proposed by Neculai Stanciu-Romania

S.2251 If ABC is an acute triangle with the medians AM, BN, CP and $\sphericalangle CBN = \alpha, \sphericalangle ACP = \beta$, $\sphericalangle BAM = \gamma$, then prove that: $i)$ $\cot\beta + \cot\gamma - \cot\alpha > \cot(C-\beta) - \cot(A-\gamma) - \cot(B-\alpha)$

$$ii)\ \cot\beta + \cot\gamma - \cot\alpha > \cot C - \cot A - \cot B$$

Proposed by Marius Drăgan, Neculai Stanciu-Romania

S.2252 Solve for positive integers $a^3 + 9b^2 + 9c^2 = 2017$, where $a \geq b \geq c$.

Proposed by George Florin Șerban, Neculai Stanciu-Romania

S.2253 If ABC is a triangle with $\sphericalangle A = 90°, \sphericalangle C = 30°, AD$ —altitude from A, I —incenter and T midpoint of BI, then prove that AT is angle bisector of $\sphericalangle BAD$.

Proposed by Neculai Stanciu-Romania

S.2254 How many divisors has the positive integer n which verify: $n^n = 2027^{2027^{2028}}$?

Proposed by Neculai Stanciu-Romania

S.2255 If x and y are positive integers with $\frac{2010}{2011} < \frac{x}{y} < \frac{2011}{2012}$, then compute the minimum value for $x + y$ and the values of x and y which achieves this minimum.

Proposed by Neculai Stanciu-Romania

S.2256 How many digits has the number $2^{320} \cdot 5^{240}$?

Proposed by Neculai Stanciu-Romania

S.2257 If ABC is right angled with $A = 90°, AD \perp BC$ and E is the intersection of the bisector of$\sphericalangle ADC$ with $AC, M \in (AE), N \in (DC), \{F\} = MN \cap AD$ such that $\frac{AE}{DC} = \frac{ME}{NC} = k > 1$, then determine the ratio $\frac{BD}{AF}$ in terms of k.

Proposed by Neculai Stanciu-Romania

S.2258 Prove that the number $5^{n+k} - 5^n$ can be written as the sum of $2k$ different non-zero perfect squares for any positive integers n and k.

Proposed by Marius Drăgan, Neculai Stanciu-Romania

S.2259 In ΔABC the following relationship holds:

$$2\max\left\{\frac{m_a}{h_a}, \frac{m_b}{h_b}, \frac{m_c}{h_c}\right\} \geq \frac{m_a}{m_b} + \frac{m_b}{m_c} + \frac{m_c}{m_a} - 1$$

Proposed by Bogdan Fuștei-Romania

S.2260 In ΔABC the following relationship holds:

$$\left(\frac{R}{r} - 1\right)\sqrt{\frac{2r}{R}} \geq \frac{\sqrt{n_a n_b + n_b n_c + n_c n_a}}{s}$$

Proposed by Bogdan Fuștei-Romania

S.2261 If $xyz(x+y+z)>0$ then:

$$\left|yz\sqrt{\frac{m_a}{h_a}}+zx\sqrt{\frac{m_b}{h_b}}+xy\sqrt{\frac{m_c}{h_c}}\right|\geq\sqrt{\left(2\sum_{cyc}\sqrt{\frac{m_am_b}{h_ah_b}}-\sum_{cyc}\frac{m_a}{h_a}\right)(x+y+z)xyz}$$

Proposed by Bogdan Fuștei-Romania

S.2262 In $\Delta ABC, I$ −incenter, the following relationship holds:

$$\sum_{cyc}\frac{1}{\sin\frac{A}{4}}\geq\frac{AI+BI+CI+s}{r}+3\left(\sqrt{2}+\sqrt{6}-\sqrt{3}-2\right)$$

Proposed by Bogdan Fuștei-Romania

S.2263 In ΔABC the following relationship holds: $n_a^2+g_a^2\geq n_ag_a+am_a-rr_a$

Proposed by Bogdan Fuștei-Romania

S.2264 In ΔABC the following relationship holds:

$$\frac{m_a+m_b+m_c}{9F^2}\geq\frac{1}{2}\left(\min\left\{\frac{1}{h_ah_bm_c},\frac{1}{h_bh_cm_a},\frac{1}{h_ah_cm_b}\right\}+\max\left\{\frac{1}{h_ah_bm_c},\frac{1}{h_bh_cm_a},\frac{1}{h_ah_cm_b}\right\}\right)$$

Proposed by Bogdan Fuștei-Romania

S.2265 In ΔABC the following relationship holds:

$$am_b+bm_c+cm_a\geq r\sqrt{4-\frac{2r}{R}}(m_a+m_b+m_c)+\frac{s(m_am_b+m_bm_c+m_cm_a)}{m_a+m_b+m_c}$$

Proposed by Bogdan Fuștei-Romania

S.2266 In $\Delta ABC, I$ −incenter and r_1,r_2,r_3 −Malfatti's radies, the following relationship holds:

$$AI+BI+CI\leq 2(r_1+r_2+r_3)+s-3r$$

Proposed by Bogdan Fuștei-Romania

S.2267 In ΔABC the following relationship holds: $\sum_{cyc}\sqrt{n_a^2+g_a^2+2w_a^2}\leq\sum_{cyc}\frac{cm_b+bm_c}{a}$

Proposed by Bogdan Fuștei-Romania

S.2268 In $\Delta ABC, r_i, i=1,2,3$ −Malfatti's radies and I −incenter, then holds:

$$\frac{r_1}{r_a}+\frac{r_2}{r_b}+\frac{r_3}{r_c}=\frac{3(s-r)-AI-BI-CI}{2s}$$

Proposed by Bogdan Fuștei-Romania

S.2269 In ΔABC the following relationship holds:

$$m_a^2+m_b^2+m_c^2=g_a^2+g_b^2+g_c^2+(R-2r)(h_a+h_b+h_c)$$

Proposed by Bogdan Fuștei-Romania

S.2270 In ΔABC the following relationship holds:

$$\prod_{cyc}[2(m_bh_c+m_ch_b)-h_a(h_b+h_c)]\geq h_ah_bh_c\prod_{cyc}(h_a+h_b)$$

Proposed by Bogdan Fuștei-Romania

S.2271 If $a,b,c>0$, then prove that

$$\frac{a}{a+b}+\frac{b}{b+c}+\frac{c}{c+a}\leq\frac{1}{2}\left(\frac{a}{c}+\frac{b}{a}+\frac{c}{b}\right)$$

Proposed by Neculai Stanciu-Romania

S.2272 If $X,Y,Z\in M_n(\mathbb{R}),n\geq 2$ with $XZ=ZX$ and $Z^2=XY+Y$, then prove that:

$$\det(X^2+YX+Y)\geq 0$$

Proposed by Marius Drăgan, Neculai Stanciu-Romania

S.2273 Determine $x>0$ which satisfy $\left\{\frac{3x+5}{x+2}\right\}+\left[\frac{3x+2}{x+1}\right]=2.(7)$, where $\{x\}$ represents the fractional part of x and $[x]$ represents the integral part of x.

Proposed by Neculai Stanciu-Romania

S.2274 Compute:

$$\Omega=\int_0^{\frac{\pi}{2}}\sin^2x\left(\cos x\cos^2\left(\frac{\pi}{2}\sin x\right)+\sin x\cos^2\left(\frac{\pi}{2}\cos x\right)\right)dx$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

S.2275 If $A=\begin{pmatrix}1 & m & -m\\ m & 1 & 0\\ m & 0 & 1\end{pmatrix}$, then compute $A^n,n\in\mathbb{N}$.

Proposed by Neculai Stanciu-Romania

S.2276 If $a,y,z\in\mathbb{R},x+y>0,y+z>0,z+x>0,xy+yz+zx>0$ then in ΔABC holds:

$$a^2x+b^2y+c^2z\geq\frac{8(x+y+z)}{a^2+b^2+c^2}F^2+\frac{(a^2+b^2+c^2)(xy+yz+zx)}{2(x+y+z)}$$

Proposed by Bogdan Fuștei-Romania

S.2277 In ΔABC, $(r_i)_{i=1,2,3}$ −Malfatti's radies, the following relationship holds:

$$2\sum_{cyc}\frac{h_a}{w_a+a-n_a-2\sqrt{r_2r_3}}\le\frac{s}{r}+\sum_{cyc}\frac{n_a}{r_a}$$

Proposed by Bogdan Fuștei-Romania

S.2278 Find:

$$\Omega=\int_0^{\frac{\pi}{2}}\frac{dx}{(1+\tan x)(1+\tan^3 x)}$$

Proposed by Asmat Qatea-Afghanistan

S.2279 Let $a,b,c>0, ab+bc+ca+2abc=1$. Prove that:

$$\sqrt{\frac{ab}{c}}+\sqrt{\frac{bc}{a}}+\sqrt{\frac{ca}{b}}+3\sqrt{abc}\ge\sqrt{a+b+abc}+\sqrt{b+c+abc}+\sqrt{c+a+abc}$$

Proposed by Phan Ngoc Chau-Vietnam

S.2280 In ΔABC the following relationship holds:

$$\frac{3}{2}+\sum_{cyc}\sqrt{h_ah_b}\ge 3\sqrt{\frac{1}{4}+6r^2\sqrt{\frac{6}{R}}}$$

Proposed by Laura and Gheorghe Molea-Romania

S.2281 Prove that the following inequality

$$\frac{a^2}{a^2+b^2}+\frac{b^2}{b^2+c^2}+\frac{c^2}{c^2+a^2}+6\ge\frac{9\sqrt[3]{abc}+2\left(\sqrt{ab}+\sqrt{bc}+\sqrt{ca}\right)}{2}$$

holds for all positive real numbers such that $a+b+c=3$.

Proposed by Phan Ngoc Chau-Vietnam

S.2282 Let P be point in plane of ΔABC the following relationship holds:

$$\sum_{cyc}AP\sqrt{a(b+c-a)}\ge\sqrt{\frac{1}{2}\sum_{cyc}a(b+c-a)(b^2+c^2-a^2)+8F^2}$$

Proposed by Bogdan Fuștei-Romania

S.2283 Prove that any real root α of the equation $x^{6n}=4x^{2n}+4, n\in\mathbb{N}^*$ verify: $|\alpha|>\sqrt[2n]{2}$

Proposed by Gheorghe Molea-Romania

S.2284 In ΔABC the following relationship holds: $(a^2b^2+s)(b^2c^2+s)(c^2a^2+s)\geq 108\sqrt{3}\cdot F^3$

Proposed by D.M. Bătinețu-Giurgiu,Mihaly Bencze

S.2285 Let $m\geq 0, M\in Int(\Delta ABC)$ such that $x=MA, y=MB, z=MC$, then:

$$(x^{2m+2}+a^{2m+2})(y^{2m+2}+b^{2m+2})(z^{2m+2}+c^{2m+2})\geq 2^{m+4}\left(\sqrt{3}\right)^{m+1}\cdot F^{3m+3}$$

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase

S.2286 In ΔABC the following relationship holds:

$$a(m_bm_c-m_a^2)+b\left(m_am_c-m_b^2\right)+c(m_am_b-m_c^2)\geq 0$$

Proposed by Bogdan Fuștei-Romania

S.2287 Let $m,n\geq 0, m+n,x,y>0$ and $A_1B_1C_1, A_2B_2C_2$ two triangles with areas F_1 respectively F_2, then holds:

$$m(a_1^2a_2^2+x)(b_1^2b_2^2+x)(c_1^2c_2^2+x)+n(a_1^2a_2^2+y)(b_1^2b_2^2+y)(c_1^2c_2^2+z)\geq 36(mx^2+ny^2)F_1F_2$$

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase-Romania

S.2288 Let $a,b,c,s,t>0$ then:

$$(a^4+(t^2+s^2)a^2+t^2s^2)(b^4+(t^2+s^2)b^2+t^2s^2)(c^4+(t^2+s^2)c^2+t^2s^2)\geq\frac{81}{16}t^4s^4(ab+bc+ca)^4$$

Proposed by D.M. Bătinețu-Giurgiu-Romania

S.2289 Let $x,y>0$ then in ΔABC holds:

$$(x^2+y^2)(a^2b^2+b^2c^2+c^2a^2)\geq 32xy\cdot F^2+\sum_{cyc}a^2(bx-cy)^2$$

Proposed by D.M. Bătinețu-Giurgiu, Florică Anastase-Romania

S.2290 In ΔABC holds: $\frac{1}{r_a^2}+\frac{1}{r_b^2}+\frac{1}{r_c^2}\geq\frac{4}{3R^2}+\frac{1}{2}\sum_{cyc}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)^2$

Proposed by D.M. Bătinețu-Giurgiu,Mihaly Bencze-Romania

S.2291 Let $x,y>0$ then in ΔABC holds:

$$(x^2+y^2)\left(m_a^2+m_b^2+m_c^2\right)\geq\frac{32\sqrt{3}xy}{3}\cdot F+\frac{4}{3}\sum_{cyc}(xa-yb)^2$$

Proposed by D.M. Bătinețu-Giurgiu, Dan Nănuți-Romania

S.2292 If $a,b,c\geq 0, a+b+c=\lambda, \lambda>0$ then holds: $\sum_{cyc}\frac{a^2+ab+b^2}{(a+b+\lambda)^2}\leq\frac{1}{2}$.

Proposed by Marin Chirciu-Romania

S.2293 In ΔABC the following relationship holds:

$$22{,}5+\sum_{cyc}\frac{\sin^5 A}{\sin^5 B+\sin^5 C}\le 24\left(\frac{R}{2r}\right)^5.$$

Proposed by Marin Chirciu-Romania

S.2294 In ΔABC the following relationship holds:

$$s_a^n s_b+s_b^n s_c+s_c^n s_a\ge\frac{162r^4}{R}\left(\frac{6r^2}{R}\right)^{n-2},n\in\mathbb{N},n\ge 2.$$

Proposed by Marin Chirciu-Romania

S.2295 If $a_1,a_2,\dots,a_n\dots>0,a_1+a_2+\dots+a_n+\dots=1$ then holds:

$$\frac{a_1}{2a_1+1}+\frac{a_2}{4a_2+1}+\dots+\frac{a_n}{2^n a_n+1}+\dots\le\frac{1}{2}.$$

Proposed by Marin Chirciu-Romania

S.2296 Let p be a prime number, fixed. Solve for positive integers: $\begin{cases}xy+z^2=2p+3\\ x+yz^2=p+3\end{cases}.$

Proposed by Marin Chirciu-Romania

S.2297 If $x,y,z>0,xy+yz+zx=1$ then holds: $\frac{1}{3x^2+2}+\frac{1}{3y^2+2}+\frac{1}{3z^2+2}\le 1.$

Proposed by Marin Chirciu-Romania

S.2298 Let $\lambda\in\mathbb{R}$ fixed. Solve for real numbers: $2^{x^2-3\lambda x}+2^{\lambda x-x^2}=2^{1-\lambda x}.$

Proposed by Marin Chirciu-Romania

S.2299 In ΔABC the following relationship holds:

$$\frac{3}{s}\left(\frac{2r}{R}\right)^{\frac{1}{3}}\le\sum_{cyc}\frac{\csc A}{m_b+m_c}\le\frac{R}{F}\left(\frac{R}{r}-\frac{1}{2}\right).$$

Proposed by Marin Chirciu-Romania

S.2300 If $a,b>0$ and $\frac{1}{5}\le\lambda\le\frac{12}{5}$ then holds:

$$\big((\lambda+1)a^2+ab+2b^2\big)\big((\lambda+1)b^2+ab+2a^2\big)\le\left(\frac{\lambda+4}{\lambda+1}\right)^2(\lambda a^2+b^2)(\lambda b^2+a^2).$$

Proposed by Marin Chirciu-Romania

All solutions for proposed problems can be finded on the http//:www.ssmrmh.ro which is the adress of Romanian Mathematical Magazine-Interactive Journal.

UNDERGRADUATE PROBLEMS

U.2031 Find all value of $\beta \geq 0$ such that:

$$\int_{-\infty}^{\infty} \frac{e^{\beta x}}{x^2+x+1} dx < +\infty$$

Proposed by Nguyen Van Canh-Vietnam

U.2032 In ΔABC, find all numbers $\lambda > 0$ such that:

$$\sum_{cyc} n_a^2 \leq \sum_{cyc} m_a^2 + \lambda(R^2 - 4r^2)$$

Proposed by Nguyen Van Canh-Vietnam

U.2033 Find all roots of the equation

$$\frac{\left(1+y+\sqrt{y(y+2)}\right)^{-1}\left(1+y+\sqrt{y(y+2)+2}\right)^{-1}}{\sqrt{y(y+2)((y+1)^2+1)}} = 1$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2034 Solve the system for a, b: $\begin{cases} \frac{a+b}{2} + \sqrt{ab} = a^2 + b^2 \\ \sqrt{a} + \sqrt{b} = \frac{1}{2}\left(\sqrt{5} - 1\right) \end{cases}$

Proposed by Srinivasa Raghava-AIRMC-India

U.2035 If we define the integral-function:

$$\psi(n) = \int_{-\infty}^{\infty} \sqrt{1+\cosh(\pi n x)}\, e^{-\frac{\pi x(x+1)}{2}} dx$$

then prove:

$$\int_{-\infty}^{\infty}\psi(n+1)\psi(n-1)e^{-\frac{\pi n(n+1)}{2}}dn=2e^{\frac{\pi}{4}}(1+e^{\pi})\left(1+e^{\frac{\pi}{4}}\right)$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2036 Prove the integral relation:

$$\int_{-\infty}^{\infty}\sin\left(\frac{\pi x}{n}\right)\cos(\pi nx)\,e^{-\frac{\pi x(x+1)}{2}}dx=(-\coth(\pi))^{(-1)^n}\int_{-\infty}^{\infty}\sin(\pi nx)\cos\left(\frac{\pi x}{n}\right)e^{-\frac{\pi x(x+1)}{2}}dx$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2037 Solve the equations for x,y: $\sqrt{x+y}+y\sqrt{11}=7;\quad \sqrt{x+y}+x\sqrt{7}=11$

Proposed by Srinivasa Raghava-AIRMC-India

U.2038 Let $f(x)=ax^4+bx^2+c,a,b,c\in\mathbb{R}$ such that

x	$-\infty$		-2		0		2		∞
$f'(x)$		$+$	0	$-$	0	$+$	0	$-$	

Find all value $m\in\mathbb{Z}$ such that $f\big(f(m)\big)=2022m$.

Proposed by Nguyen Van Canh -Vietnam

U.2039 In ΔABC the following relationship holds:

$$\sqrt{\frac{b+c}{a}}+\sqrt{\frac{a+b}{c}}+\sqrt{\frac{a+c}{b}}+\sqrt{\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}}\le\left(3\sqrt{2}+\frac{\sqrt{6}}{2}\right)\frac{R}{2r}$$

Proposed by Nguyen Van Canh-Vietnam

U.2040 Find:

$$\Omega=\int_0^{\infty}\frac{\log(1+x)}{x(x+1)(x^2+1)}dx$$

Proposed by Vasile Mircea Popa-Romania

U.2041 Prove the integral identity:

$$\int_0^{\infty}\frac{\left(1+y+\sqrt{y(y+2)}\right)^{-2}\left(1+y+\sqrt{y(y+2)+2}\right)^{-2}}{\sqrt{y(y+2)((y+1)^2+1)}}dy=\frac{2}{3}(\sqrt{2}-2)-\frac{\Gamma\left(\frac{1}{4}\right)^2+6\Gamma\left(\frac{5}{4}\right)\Gamma\left(-\frac{3}{4}\right)}{12\sqrt{2}\pi}$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2042 If M lies on incircle of ΔABC: $\frac{MA^2}{r_a}+\frac{MB^2}{r_b}+\frac{MC^2}{r_c}=4R-3r$

Proposed by Marin Chirciu-Romania

U.2043 Find $x\in\mathbb{R}$ so that the double inequality

$$\left(\frac{R}{r}\right)^x \geq 3+\cot^x A+\cot^x B+\cot^x C \geq \left(\frac{b}{c}+\frac{c}{b}\right)^x$$

holds in any ΔABC.

Proposed by Alex Szoros-Romania

U.2044 Solve for real numbers: $\sqrt{7x^2+69x+13}+\sqrt{7x^2+55x+13}=7x$

Proposed by Marin Chirciu-Romania

U.2045 Prove:

$$\int_0^\infty \int_0^{\frac{\pi}{2}} \frac{x}{y}\cdot\frac{\sin y}{\sin x}dxdy=\pi\beta(2)$$

Proposed by Ankush Kumar Parcha-India

U.2046 If $a,b,c>0$ prove that:

$$\sum_{cyc}\frac{b+c}{\sqrt{a}}-\sum_{cyc}\sqrt{2(b+c)} \geq \frac{1}{4(a+b+c)\sqrt{abc}}\sum_{cyc}c(a-b)^2$$

Proposed by Neculai Stanciu-Romania

U.2047 Let $a,b,c\geq 0, a^2+b^2+c^2=1$. Prove that:

$$\frac{a^4-a^2}{bc-1}+\frac{b^4-b^2}{ca-1}+\frac{c^4-c^2}{ab-1}\leq ab+bc+ca$$

Proposed by Phan Ngoc Chau-Vietnam

U.2048 Solve for real numbers:

$$e^{ix}+e^{2ix}+e^{3ix}+\frac{1}{e^{ix}}+\frac{1}{e^{2ix}}+\frac{1}{e^{3ix}}=-1$$

Proposed by Daniel Sitaru,Gilena Dobrică-Romania

U.2049 If $x,y,z>0$ then:

$$\sum_{cyc}\frac{\operatorname{erf}^4(x+y)}{\operatorname{erf}^3(x)}+\sum_{cyc}\operatorname{erf}^4(x)\operatorname{erf}(y)\geq 2(\operatorname{erf}(x)+\operatorname{erf}(y)+\operatorname{erf}(z))$$

Proposed by Daniel Sitaru,Sabina Subțirelu-Romania

U.2050 If $a,b,c,d,e,f>0, a+b+c=d+e+f=1$ then:

$$\sqrt[5]{ad^4}+\sqrt[5]{be^4}+\sqrt[5]{cf^4}\leq 1$$

Proposed by Daniel Sitaru,Cristina Ene-Romania

U.2051 Find:

$$\Omega=\lim_{n\to\infty}\frac{\sqrt{H_1}+\sqrt{\frac{1}{2}H_2}+\sqrt{\frac{1}{3}H_3}+\cdots+\sqrt{\frac{1}{n}H_n}}{n\sqrt{H_n(H_1+H_2+\cdots+H_n)}}$$

Proposed by Daniel Sitaru,Iacob Meda-Romania

U.2052 If $a,b,c,d>0$ then: $\frac{(a^3+b^3)(a^4+b^4+c^4)(a^5+b^5+c^5+d^5)}{a^3b^3c^2d(a+b)(a+b+c)(a+b+c+d)}\geq 1$

Proposed by Daniel Sitaru,Alina Tigae-Romania

U.2053 If $m,n,p\in\mathbb{N}-\{0\}$ then:

$$\sum_{k=1}^{n}\frac{1}{k}+\sum_{k=1}^{m}\frac{1}{k}+\sum_{k=1}^{p}\frac{1}{k}\leq 2+\sum_{k=1}^{mnp}\frac{1}{k}$$

Proposed by Daniel Sitaru,Elena Grigore-Romania

U.2054

$$M=\left\{\begin{pmatrix}a & b & c\\ \bar{b} & a & d\\ \bar{c} & d & a\end{pmatrix}\middle|\, a>0, b,c,d\in\mathbb{C}\right\}$$

If $X,Y,Z,T\in M$ then: $\sqrt[3]{\det(X+Y+Z+T)}\geq\sqrt[3]{\det X}+\sqrt[3]{\det Y}+\sqrt[3]{\det Z}+\sqrt[3]{\det T}$

Proposed by Daniel Sitaru,Elena Alexie-Romania

U.2055 If $n\in\mathbb{N}, n\geq 4$ then: $\frac{(1+n^2)^{n+1}-1}{n+1}\geq n^2+2n^4+2n^6+n^8+\frac{1}{5}n^{10}$

Proposed by Daniel Sitaru,Mihaela Dăianu-Romania

U.2056 Find:

$$\Omega=\int\frac{\sin x+4\cos x}{5(e^{-x}+\sin x)+3\cos x}dx$$

Proposed by Daniel Sitaru,Simona Radu-Romania

U.2057 Solve for real numbers:

$$\begin{vmatrix}-\varphi & \varphi-\frac{2}{\sqrt{\varphi}} & \varphi\\ \sqrt{\varphi} & -\sqrt{\varphi} & \sqrt{\varphi}-\frac{2}{\sqrt{\varphi}}\\ 1-2x\sqrt{\varphi} & 1 & -1\end{vmatrix}=0;\ \varphi-\text{golden ratio.}$$

Proposed by Daniel Sitaru,Claudiu Ciulcu-Romania

U.2058 Prove:

$$\sum_{n=0}^{\infty}\sum_{m=0}^{\infty}(-1)^{n+m}\beta(n+2,1)\beta(1,m+2)H_{\left[\frac{n+2}{2}\right]}H_{\left[\frac{m+2}{2}\right]}=\log^4 2$$

where $[\cdot]$ −floor function.

Proposed by Ankush Kumar Parcha-India

U.2059 Prove that:

$$\int_{\infty}^{1}\frac{1+y}{y(1+y^2)}\log\left(\frac{y-1}{y+1}\right)dy=\beta(2)+\frac{3}{8}\zeta(2)$$

where $\zeta(s)$ −Euler-Riemann zeta function and $\beta(x)$ −Dirichlet-beta function.

Proposed by Ankush Kumar Parcha-India

U.2060 In ΔABC the following relationship holds:

$$ab+bc+ca\geq 4F\sqrt{\frac{3}{1-k}};k=\frac{4r^2(R-2r)}{27R^2(R-2r)}$$

Proposed by Adil Abdullayev-Azerbaijan

U.2061 In ΔABC the following relationship holds:

$$\frac{a^4+b^4+c^4}{a^2b^2+b^2c^2+c^2a^2}+\frac{s_as_bs_c}{m_am_bm_c}\geq 2$$

Proposed by Adil Abdullayev-Azerbaijan

U.2062 In ΔABC the following relationship holds:

$$\left(\frac{(a+b)(b+c)(c+a)}{8abc}\right)^3\leq\frac{R}{2r}$$

Proposed by Adil Abdullayev-Azerbaijan

U.2063 If $0<a\leq b, m>0$ then:

$$b-a+\frac{m^2}{e(m+1)\left(b^{1+\frac{1}{m}}-a^{1+\frac{1}{m}}\right)}\geq\log\left(\frac{b^b}{a^a}\right)$$

Proposed by Daniel Sitaru,Mirea Mihaela Mioara-Romania

U.2064 In ΔABC the following relationship holds:

$$m_a^2=\left(\frac{b\cos B-c\cos C}{2}\right)^2+\left(\frac{b^2+c^2}{4R}\right)^2$$

Proposed by Adil Abdullayev-Azerbaijan

U.2065 Prove that:

$$\int_0^1\frac{z^2\log\left(\frac{1}{z}\right)}{(1-z^4)\left(\pi^2+\left(\log\left(\frac{1+z}{1-z}\right)+2\tan^{-1}\left(\frac{1}{z}\right)\right)^2\right)}dz=\frac{1}{32}\log\left(\frac{\pi^2}{8}\right)$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2066 In ΔABC prove or disprove:

$$\sum_{cyc}g_a^2+2\sum_{cyc}w_a^2+3\sum_{cyc}n_a^2\ge 6s^2,\qquad 3\sum_{cyc}g_a^2+2\sum_{cyc}w_a^2+\sum_{cyc}n_a^2\le 6s^2,\qquad g_a+2m_a>n_a$$

Proposed by Nguyen Van Canh-Vietnam

U.2067 Find a closed form:

$$\Omega=\int_0^1\int_0^1\frac{dxdy}{(x+y)\sqrt{(1-x)(1-y)}}$$

Proposed by Abdul Mukhtar-Nigeria

U.2068 If $0<a\le b$ then:

$$\int_a^b\int_a^b\int_a^b(x+y+z)\left(\frac{x^2}{y}+\frac{y^2}{z}+\frac{z^2}{x}\right)dxdydz\ge 3(b-a)^3(a^2+ab+b^2)$$

Proposed by Daniel Sitaru,Roxana Vasile-Romania

U.2069 Let $u_1=a\in\mathbb{R}, u_{n+1}=2023u_n^4+u_n-2022, \forall n\in\mathbb{N}, n\ge 1$. Find all value of a such that

$$\lim_{n\to\infty}u_n\in\mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

U.2070 Prove that:

$$\int_0^{\frac{\pi}{4}}\tanh^{-1}\left(\frac{\csc x-\sec x}{\csc x+\sec x}\right)dx=\frac{G}{2}$$

Proposed by Ankush Kumar Parcha-India

U.2071 Prove that:

$$\frac{4}{3\pi}\int_0^{\frac{\pi}{2}}\frac{\sqrt{2}(1+\cos x\sqrt{(1+\sin^2 x)\sqrt{1+\sqrt{1-\sin^4 x}}}-4}{\cos^2 x\,(1+\sin^2 x)-1}dx=\frac{20}{9\pi}+\frac{\Gamma^2\left(\frac{1}{4}\right)}{9\pi\sqrt{2\pi}}+\frac{4\sqrt{2\pi}}{3\Gamma^2\left(\frac{1}{4}\right)}$$

Proposed by Naren Bhandari-Bajura-Nepal

U.2072 Find:

$$\Omega=\int_0^{\frac{\pi}{2}}\frac{\sin^{-1}(\sin^2 x)}{\sin x}dx$$

Proposed by Naren Bhandari-Bajura-Nepal

U.2073 Find:

$$\Omega=\lim_{n\to\infty}\left(\int_0^{\infty}\frac{x^{2n}}{e^{x^2}}dx\right)^{\frac{1}{n}}\left(\int_0^{\infty}\frac{x^{2n+1}}{e^{x^2}}dx\right)^{-\frac{1}{n}}$$

Proposed by Daniel Sitaru, Lavinia Trincu-Romania

U.2074 Find:

$$\Omega=\int\left(\frac{\log_2^2 x}{x}\right)^2\left(\log_2\left(\frac{2^x}{x}\right)\right)^{-3}dx$$

Proposed by Daniel Sitaru,Ileana Stanciu-Romania

U.2075 If $x,y,z\in\left[0,\frac{\pi}{2}\right]$ then:

$$\exp\left(\frac{2}{3}\sum_{cyc} sin\,x\,(4-cos^2\,x)\right)\le\prod_{cyc}\left(\frac{1+\sin x}{1-\sin x}\right)$$

Proposed by Daniel Sitaru, Dan Mitricoiu-Romania

U.2076 Find all $x,y,z\in\left(0,\frac{\pi}{2}\right]$ such that:

$$\frac{1}{1+\sin x+\sin y+\sin z}+\sum_{cyc}\frac{1}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^4}=1$$

Proposed by Daniel Sitaru, Maria Lavinia Popa-Romania

U.2077 If $a,b,c>0$ then:

$$\sum_{cyc}\log\left(\frac{c+a}{c}\right)\log\left(\frac{c+b}{c}\right)<\sum_{cyc}\left(\frac{a}{b}\right)^{2022}$$

Proposed by Daniel Sitaru, Sorin Pîrlea-Romania

U.2078 Solve for real numbers:

$$\frac{1}{x}\left(e^{\frac{\pi}{2}}+\int_1^x \frac{(t+1)^2}{t^2+1}\cdot e^{2\tan^{-1}t}dt\right)=e^{\frac{2\sqrt{3}}{3}}$$

Proposed by Daniel Sitaru, Dorina Goiceanu-Romania

U.2079 If $x\geq 1$ then:

$$4x^2(1+x)\left(1+\frac{1}{x}\right)^{\frac{1}{x}}\leq (2+x+x^2)^2$$

Proposed by Daniel Sitaru, Iulia Sanda-Romania

U.2080 Solve for real numbers: $5^{x^3+26x}+x^6+244x^2=5^{9x^2+24}+29x^4+576$

Proposed by Daniel Sitaru,Nicolae Radu-Romania

U.2081

$$\Omega(a)=\int_{-a}^{a}\cos^{-1}(x-x^3+x^5-x^7+x^9)\,dx\,,a>0$$

$$\omega_1=\sum_{cyc}(\Omega(a)+\sqrt{\big(\Omega(a)+\Omega(b)\big)\big(\Omega(a)+\Omega(c)\big)}\,,\omega_2=\sum_{cyc}\Omega(a)\Omega(b)\cdot\sum_{cyc}\frac{1}{\sqrt{\Omega(a)\Omega(b)}}$$

Prove that: $\omega_1\leq\omega_2$.

Proposed by Daniel Sitaru, Mihaela Stăncele-Romania

U.2082 If $a,b>0,n\in\mathbb{N}^*$ then:

$$(a^2+b^2)^n\cdot\log^n\left(\frac{a^2+b^2}{a+b}\right)\leq(a+b)^{n-1}(a^{n+1}\log^n a+b^{n+1}\log^n b)$$

Proposed by Daniel SitaruMihai Ionescu-Romania

U.2083

$$x,y,z>0,x+y+z=3,A,B,C\in M_n(\mathbb{R}),n\in\mathbb{N}^*,\big(N_F(A)\big)^2=\sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}^2$$

Prove that:

$$\frac{x^2+y^2+z^2}{N_F(A+B+C)}\geq\frac{3}{N_F(A)+N_F(B)+N_F(C)}$$

Proposed by Daniel Sitaru,Alecu Orlando-Romania

U.2084 Prove that:

$$\cos^7\left(\frac{\pi}{18}\right)+\cos^9\left(\frac{5\pi}{18}\right)+\cos^9\left(\frac{7\pi}{18}\right)=\cos^9\left(\frac{\pi}{18}\right)+\cos^7\left(\frac{5\pi}{18}\right)+\cos^7\left(\frac{7\pi}{18}\right)$$

Proposed by Carlos Paiva-Brazil

U.2085 Find:

$$\Omega=\int_0^1\int_0^1\frac{xy^3\log x\log y}{(1-x^2y^2)(1-x^2)(1-y^2)}dxdy$$

Proposed by Narendra Bhandari-Nepal

U.2086 Find:

$$\Omega=\int_0^1\left(\frac{1}{1-x^2}+\psi_0(1-x^2)\right)dx$$

where $\psi_0(z)$ is digamma function.

Proposed by Naredra Bhandari-Nepal

U.2087 Let $a,b,c\geq 0, ab+bc+ca=1$. Prove that:

$$\frac{\sqrt{3ab+1}}{ab+c}+\frac{\sqrt{3bc+1}}{bc+a}+\frac{\sqrt{3ca+1}}{ca+b}\geq 4$$

Proposed by Phan Ngoc Chau-Vietnam

U.2088 Solve the integral for n:

$$\int_{-\infty}^{\infty}(-1)^x e^{-\frac{\pi x}{2}(nx+1)}dx=\frac{1}{\sqrt{n}}$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2089 If we have the integral relation:

$$\int_{-\infty}^{\infty}\left(L_x\left[\left(e^{-x^2-xy}+e^{-x^2+xy}\right)^2\right](y)\right)e^{-ny^2}\,dy=\frac{1}{\sqrt{n}}$$

then find the value of the expression: $\sqrt{\sqrt{2048n^4-2816n^3+608n^2-36n}}$

Here $L_x[*]$ —is Laplace transform.

Proposed by Srinivasa Raghava-AIRMC-India

U.2090 If we have the function

$$S(n)=\sum_{m=1}^{n}\left(\left\lceil\frac{m}{2}\right\rceil(-1)^{\left\lfloor\frac{m}{2}\right\rfloor}+(-1)^{\left\lceil\frac{m}{2}\right\rceil}\left\lfloor\frac{m}{2}\right\rfloor\right)\left(\sin\left(\frac{\pi m}{2}\right)+\cos\left(\frac{\pi m}{2}\right)\right)$$

then prove the relation

$$(-1)^{S(n)} = 1 + n(2n^2 + n + 3) - 4\left\lceil\frac{1}{4}(n(2n^2 + n + 3) + 2)\right\rceil$$

where $\lfloor * \rfloor$ is floor function and $\lceil * \rceil$ is ceiling function.

Proposed by Srinivasa Raghava-AIRMC-India

U.2091 In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{h_a}{\sqrt{h_b^2 - h_b h_c + h_c^2}} \ge \frac{12r^2}{R^2}$$

Proposed by Marian Ursărescu-Romania

U.2092 Find a closed form:

$$\Omega(a) = \int_0^\infty \frac{x}{(x+1)(x^2+1)(1+a^2x^2)} dx\,, a \in \mathbb{R}$$

Proposed by Vasile Mircea Popa-Romania

U.2093 Prove the summation:

$$\sum_{n=0}^{\infty} \frac{mod(4n^3 + 5n + 1, 4n + 1)}{n!} = \frac{1}{8}\left(\frac{7}{e} + e - 2(\sin(1) + 7\cos(1))\right)$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2094 Given $A_0 = 1$ and $A_{n+1} = (n-1) + (n+2)A_n$ then prove that: $A_n \sim (6 - 2e) \cdot (n+1)!$

Proposed by Amrit Awasthi-India

U.2095 In ΔABC the following relationship holds:

$$\frac{1}{F}\sum_{cyc} m_a m_b \ge \frac{3}{2}\sqrt{\left(\frac{m_a}{m_b}\right)^2 + \left(\frac{m_b}{m_c}\right)^2 + \left(\frac{m_c}{m_a}\right)^2} + \sqrt{\sum_{cyc} \frac{m_a(m_b + m_c - m_a)}{m_b(m_a + m_c - m_b)}}$$

Proposed by Bogdan Fuştei-Romania

U.2096 In ΔABC the following relationship holds:

$$\frac{ab + bc + ca}{2F} \ge \sqrt{\left(\frac{a}{b}\right)^2 + \left(\frac{b}{c}\right)^2 + \left(\frac{c}{a}\right)^2} + \sqrt{\frac{a(s-a)}{b(s-b)} + \frac{b(s-b)}{c(s-c)} + \frac{c(s-c)}{a(s-a)}}$$

Proposed by Bogdan Fuştei-Romania

U.2097 $xy + yz + Zx > 0, x + y, y + z, z + x > 0$. In ΔABC the following relationship holds:

$$\sum_{cyc} m_a(m_b+m_c)x \geq 6F\sqrt{xy+yz+zx}$$

Proposed by Bogdan Fuștei-Romania

U.2098 Find all values $\alpha \geq 0$ such that

$$\int_0^{\infty} \log(1+\alpha x^2)\,dx < +\infty$$

Proposed by Nguyen Van Canh-Vietnam

U.2099 Find all values $\beta \geq 0$ such that:

$$\int_{-\infty}^{\infty} 2021^{\beta x} x dx < +\infty$$

Proposed by Nguyen Van Canh-Vietnam

U.2100 Let $a \geq b \geq c > 0$. Prove that:

$$\sqrt[3]{\frac{a^2+bc}{b^2+c^2}} + \sqrt[3]{\frac{b^2+ca}{c^2+a^2}} \geq \max\left\{2, \sqrt[3]{\frac{4(a^2+b^2)}{c^2+ab}}\right\}$$

Proposed by Nguyen Van Canh-Vietnam

U.2101 Let $a,b,c > 0, a+b+c=1$. Prove that:

$$27(a^2+b^2)^2(b^2+c^2)^2(c^2+a^2)^2 \geq \left(2\sum_{cyc} b^2c^2 + 2bac\right)^3$$

Proposed by Nguyen Van Canh-Vietnam

U.2102 Find all functions $f: \mathbb{Z} \to \mathbb{Z}$ such that

$$6f\left(\min\left\{\frac{x+y}{2}, \frac{x-y}{2}, 0\right\}\right) + 4f\left(\max\left\{\frac{x+y}{2}, \frac{x-y}{2}, 0\right\}\right) = 10f(xy), \forall x,y \in \mathbb{Z}$$

Proposed by Nguyen Van Canh-Vietnam

U.2103 Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that

$$f(2x+1) = 3f\left(\frac{x+1}{4x^2+1}\right) + 3x - 2x^2, \forall x \in \mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

U.2104 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{a^2}{b+c}\leq\frac{1}{2}\sum_{cyc}a+\frac{R^2-4r^2}{r}$$

Proposed by Nguyen Van Canh-Vietnam

U.2105 Find all values $\alpha>0$ such that: $2021^{\alpha x}+2020^{-\alpha x}\leq 2;\forall x\in[-2,2]$

Proposed by Nguyen Van Canh-Vietnam

U.2106 Let $a,b,c>0$ such that $abc=1$ and $1\leq\alpha\leq 4$. Prove that:

$$\frac{1}{\alpha+a+b}+\frac{1}{\alpha+b+c}+\frac{1}{\alpha+c+a}\leq\frac{3}{2+\alpha}$$

Proposed by Nguyen Van Canh-Vietnam

U.2107 In ΔABC the following relatinship holds:

$$\frac{a^3}{b}+\frac{b^3}{c}+\frac{c^3}{a}+\frac{R^5}{8r^3}\geq\frac{3(a^4+b^4+c^4)}{a^2+b^2+c^2}+\frac{4(a^3+b^3+c^3)}{a+b+c}$$

Proposed by Nguyen Van Canh-Vietnam

U.2108 Let $a,b,c>0$. Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$3f(ax^3-by^3)=2f(cx^2-ay^2)+abc(x^2-y^2)f(bx^5-cy^5),\forall x,y\in\mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

U.2109 Find all functions $f:\mathbb{Z}\to\mathbb{Q}$ such that:

$$f(x-y+xy)+f(x+y-xy)=2f(x^2+y^2),\forall x,y\in\mathbb{Z}$$

Proposed by Nguyen Van Canh-Vietnam

U.2110 Let $a,b,c>0,abc=1$. Prove that:

$$a)\ \sqrt{\frac{a+b}{a+c}}+\sqrt{\frac{b+c}{b+a}}+\sqrt{\frac{c+a}{c+b}}\leq a+b+c,\qquad b)\ \sqrt[3]{\frac{a+b}{a+c}}+\sqrt[3]{\frac{b+c}{b+a}}+\sqrt[3]{\frac{c+a}{c+b}}\leq a+b+c$$

Proposed by Nguyen Van Canh-Vietnam

U.2111 In ΔABC the following relationship holds: $\left(\sum_{cyc}\sqrt[3]{\sin\frac{A}{2}}\right)^3\geq 27\sqrt{\frac{r}{2R}}$

Proposed by Nguyen Van Canh-Vietnam

U.2112 Let $x,y,z>0,xy+yz+zx+2xyz=1$. Prove that: $x+y+z\geq 2-\sqrt{2xyz}$

Proposed by Nguyen Van Canh-Vietnam

U.2113 Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that:

$$\max\{f(x),y\}+\max\{f(y),x\}=3\max\{f(x),f(y)\},\forall x,y\in\mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

U.2114 Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$f(\min\{x,y,0\})+f(\max\{x,y,0\})=2\min f(\max\{x,y,0\}+\min\{x,y,0\}),\forall x,y\in\mathbb{R}$$

Proposed by Nguyen Van Canh-Vietnam

U.2115 In ΔABC the following relationship holds:

$$F\sum_{cyc}\frac{1}{h_a}\cot\frac{A}{2}\geq 4R+R$$

Proposed by Marian Ursărescu-Romania

U.2116 Find:

$$\Omega=\int_0^\infty\frac{\tan^{-1}x}{x(x^2-x+1)}dx$$

Proposed by Vasile Mircea Popa-Romania

U.2117 In an acute triangle with r_1,r_2,r_3 −exradii, R_A,R_B,R_C −radii of circles with centers A,B,C prove that:

$$\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\geq\frac{1}{R_A+R_B+R_C}$$

Proposed by Marian Ursărescu-Romania

U.2118 Prove that:

$$\int_0^1\int_0^1\int_0^1\sqrt{x^2+y^2+z^2}\tan^{-1}\left(\sqrt{x^2+y^2+z^2}\right)dxdydz=\frac{\pi\sqrt{3}}{8}+\frac{\pi}{2}\log\left(2\cos\frac{\pi}{12}\right)-\frac{\eta(2)+3}{4}$$

Proposed by Asmat Qatea-Afghanistan

U.2119 If $m\in(0,1),m<n$ then:

$$\int_{-1}^1\sqrt{\frac{1+z}{1-z}}\log\frac{1+2mnz+(m^2+n^2-1)z^2}{1-2mnz+(m^2+n^2-1)z^2}\frac{dz}{z}=4\pi\sin^{-1}m$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2120 Find:

$$J=\lim_{m\to\sqrt{\pi}}\int_{\sqrt{\pi}}^{m}\frac{(m^2+\sqrt{\pi}z)e^{z^2}}{(m^3-\sqrt{\pi}m^2+\pi m-\pi\sqrt{\pi})z^2\log z}dz$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2121 Prove the integral:

$$\int_0^{\pi}\min\left(\sin 2x,\cos 2x,\sin\frac{x}{2},\cos\frac{x}{2}\right)dx=\frac{1}{8}\left(12-\sqrt{2\left(25\sqrt{5}+40\sqrt{\sqrt{5}+5}+141\right)}\right)$$

Proposed by Srinivasa Raghava-AIRMC-India

U.2122 Prove that: $\sum_{cyc}\frac{(2a)^2}{(s-b)(s-c)\left(3(b+c)\right)^2}\ge\frac{1}{r(4R+r)}$

occurs in any triangle ABC with usual notations.

Proposed by Neculai Stanciu-Romania

U.2123 Find:

$$\Omega=\int_0^1\int_0^1\tan^{-1}(xy)\log(1-xy)\,dxdy$$

Proposed by Togrul Ehmedov-Azerbaijan

U.2124 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{r_a}{\sqrt{r_b^2-r_br_c+r_c^2}}\ge\frac{4R+r}{4R-5r}$$

Proposed by Marian Ursărescu-Romania

U.2125 Find:

$$\Omega=\lim_{n\to\infty}\sum_{k=1}^{n}\left[\sum_{i=1}^{k}\left(i+\frac{3}{4}\right)\right]^{-1}$$

Proposed by Vasile Mircea Popa-Romania

U.2126 If $f(x)=\sqrt{x+1}, g(x)=\log(f(x)-1), h(x)=\log x, \forall x>0$, then show that:

$$\lim_{x\to 0_+}\left(-9g^2(x)+(18h(x)-48)g(x)-9f^3(x)h^2(x)+\left(12f^3(x)+3f(x)\right)h(x)\right)=$$

$$=48\log 2-9\log^2 2$$

Proposed by Narendra Bhandari-Nepal

U.2127 Find a closed form: $\Omega = \int_0^\infty \frac{e^{-x}\log\left(\frac{1}{x}\right)\sin(\alpha x)}{x}dx, \forall\alpha > 0$

Proposed by Ose Favour-Nigeria

U.2128 Let the function g be non-negative and monotone on $[0,1]$. Assuming that $\int_0^1 g(t)\,dt = 1$. Then for every pair of real numbers p, r such that $r \leq p$, we have:

$$\left(2(1+r)\int_0^1\int_0^{1-x} x^r g(x+t)g(t)\,dtdx\right)^{\frac{1}{r}} \leq \left(2(1+p)\int_0^1\int_0^{1-x} x^p g(x+t)g(t)\,dtdx\right)^{\frac{1}{p}}$$

Proposed by Minh Vu-Vietnam

U.2129 Prove that:

$$\sum_{n=1}^{\infty}\frac{(n+1)\log n}{(-1)^n n^3} = \frac{\zeta'(2)}{2} + \frac{3\zeta'(3)}{4} + \frac{\zeta(3)\log 8}{12} + \frac{\pi^2\log 2}{12}$$

Proposed by Tobi Joshua-Nigeria

U.2130 Find:

$$\Omega = \int_0^1 \frac{\log x\log(1-x)\log(1+x)}{x}dx$$

Proposed by Togrul Ehmedov-Azerbaijan

U.2131 Solve for real numbers: $\tan^5(2x) = \tan(10x)$

Proposed by Togrul Ehmedov-Azerbaijan

U.2132 Prove that:

$$\gamma = \lim_{n\to\infty}\frac{(-1)^{n+1}n^{n+1}}{n!}\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right)\Big|_{x=n}$$

where γ is Euler-Mascheroni constant.

Proposed by Amrit Awasthi-India

U.2133 For $n \in \mathbb{N}$ prove that: $H_n + \log H_n \geq \log 2 + \log n,\ \ H_n \geq \frac{2n}{n+1}$.

Proposed by Amrit Awasthi-India

U.2134 Let $a, b, c > 0, a + b + c = 3$. Prove that:

$$\frac{a^2b^2 + b^2c^2 + c^2a^2}{3} + \sum_{cyc}\left(\frac{a^2+b^2}{a+b}\right)^2 \geq 4$$

Proposed by Choy Fai Lam-Hong Kong

U.2135 Let $a, b, c > 0$. Prove that:

$$\frac{a^{3n}}{b^n}+\frac{b^{3n}}{c^n}+\frac{c^{3n}}{a^n}\ge\frac{1}{3^n}\sum_{cyc}\left(\frac{8a^3+b^3}{2a+b}\right)^n$$

Proposed by Choy Fai Lam-Hong Kong

U.2136 Given $\cot^3 x+(3-\cot^3 x)^2=3$ then prove:

$$x=\cot^{-1}\left(-\frac{1}{2}\pm\sqrt{\frac{1}{4}+\sqrt{\frac{8}{3}}}\right)+2k\pi$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2137 Find:

$$\Omega=\int_0^1\sqrt{\frac{1-z}{z(1+z)}}\log\frac{\sqrt{2z}+\sqrt{1+z}}{1+\sqrt{z}}\,dz$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2138 Find a closed form:

$$\Omega=\sum_{n\ge 0}\frac{\binom{2n}{n}^2}{2^{4n}\cdot(2n+1)}\sum_{m=1}^{n}\left(\frac{(2m+1)^2}{(m+1)^4}\cdot\frac{2^{8(m+1)}}{\binom{2m+1}{m+1}^4}-\frac{(2m-1)^2}{m^4}-\frac{2^{8m}}{\binom{2m}{m}^4}\right)$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2139 If $\frac{dy}{dx}+\frac{1}{x+(1-y)^{n-1}e^{-y}}=0, y(0)=1$ and $y=f_n(x)$ then find:

$$\Omega=\lim_{n\to\infty}\left(n^2\int_0^{\frac{1}{n}}f_n(x)\,dx\right)^n$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2140 Prove that:

$$\lim_{n\to\infty}\int_0^{\infty}\tan^{-1}\frac{2nx}{(n^2+m^2)x^2-1}\log\frac{(nx)^2+(1-mx)^2}{(nx)^2+(1+mx)^2}\,dx=2\pi m\log 2$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2141 If $0<z<0.5, f(x)=\cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x)=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ then find:

$$\Omega = \lim_{x\to z}\frac{f(x)-f(z)}{g(x)-g(z)}$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2142 Prove that:

$$\int_0^1 \frac{\log\left(x+\sqrt{1+x^2}\right)}{x}dx = \log\left(\frac{1}{2}\right)\log\left(\frac{1}{\sqrt{2}+1}\right) - \frac{\pi^2}{24} - 2Li_2\left(-\frac{1}{1+\sqrt{2}}\right)$$

where $Li_2(z)$ is dilogarithm function.

Proposed by Narendra Bhandari-Nepal

U.2143 Find:

$$\Omega = \int_0^1 \frac{\log(1+x^7)}{1+x^7}dx$$

Proposed by Daniel Immarube-Nigeria

U.2144 Let $a,b,c \geq \frac{2}{3}, a+b+c=9$. Prove that: $\sqrt{ab+bc+ca} \leq \sqrt{a}+\sqrt{b}+\sqrt{c}$

Proposed by Phan Ngoc Chau-Vietnam

U.2145 a,b,c –sides in $\Delta ABC, \sqrt{a}, \sqrt{b}\sqrt{c}$ –sides in $\Delta A'B'C'$. Prove that:

$$a\sin A' + b\sin B' + c\sin C' \leq 3F'\sqrt{\frac{2R}{r}}$$

Proposed by Mehmet Șahin-Turkiye

U.2146 In ΔABC the following relationship holds:

$$\frac{(2R-r)^2}{4r^2} \geq \left(\sum_{cyc}\frac{a}{b+c}\right)\left(\sum_{cyc}\frac{h_a}{h_b+h_c}\right)$$

Proposed by Alex Szoros-Romania

U.2147 Find:

$$\Omega = \int_0^1\int_0^1 \frac{x(\tan^{-1}x)^2}{1+xy}dxdy$$

Proposed by Togrul Ehmedov-Azerbaijan

U.2148 Prove that:

$$\lim_{n\to\infty} e^{H_n} \frac{n}{\log^2 n} \int_0^{\frac{1}{n^2}} \frac{\log(nx)\log\left(\frac{x}{n}\right)}{1+nx} dx = 3e^{\gamma}$$

Proposed by Mokhtar Khassani-Algeria

U.2149 In ΔABC the following relationship holds:

$$\frac{4R+r}{6}\sqrt{\frac{4R+r}{r}} \geq \sum_{cyc} \frac{h_a^2}{w_a+m_a} \geq \frac{r^2}{R}$$

Proposed by Mokhtar Khassani-Algeria

U.2150 For $a > b > 0$ find:

$$\Omega = \int_0^{\infty} \frac{e^{ax}-e^{bx}}{x(e^{ax}+1)(e^{bx}+1)} dx$$

Proposed by Abdul Mukhtar-Nigeria

U.2151 Find:

$$\Omega = \int_0^{\frac{1}{2}} \frac{\log x \log^2(1-x)}{x(1-x)} dx$$

Proposed by Precious Itsuokor-Nigeria

U.2152 Find:

$$\Omega = \int_0^{1} \frac{\log(1+x^2)\log\left(\sin\sqrt{x}\right)}{1+x^2} dx$$

Proposed by Precious Itsuokor-Nigeria

U.2153 If $a, b, c, d > 1$ then holds:

$$\sum_{cyc} \log_a \frac{b^{n+1}+c^{n+1}+d^{n+1}}{b^n+c^n+d^n} \geq 4, n \in \mathbb{N}.$$

Proposed by Marin Chirciu-Romania

U.2154 In ΔABC the following relationship holds:

$$\frac{2}{r} \leq \sum_{cyc} \frac{r_b+r_c}{h_a^2} \leq \frac{R}{r^2}.$$

Proposed by Marin Chirciu-Romania

U.2155 In ΔABC the following relationship holds:

$$\frac{1}{F^2} \le \sum_{cyc} \frac{1}{w_a^4} \le \frac{1}{F^2}\left(\frac{R}{2r}\right)^5.$$

Proposed by Marin Chirciu-Romania

U.2156 Let $a > 1, b > 1$ fixed. Solve for real numbers:

$$a^{\log_{2b}\left(x+\frac{b^2}{x}\right)} = \frac{(a+2b)x - b^2 - x^2}{x}.$$

Proposed by Marin Chirciu-Romania

U.2157 In ΔABC, G −centroid, x, y, z altitudes from G to BC, CA, AB, respectively. Prove that:

$$\frac{54}{F} \le \frac{b+c-a}{x^3} + \frac{c+a-b}{y^3} + \frac{a+b-c}{z^3} \le \frac{27}{F} \cdot \frac{R}{r}.$$

Proposed by Marin Chirciu-Romania

U.2158 In ΔABC the following relationship holds:

$$\frac{9R}{4r} = \frac{3}{2} \le \sum_{cyc} \frac{m_a^2}{h_b h_c} \le \frac{R^2}{r^2} + \frac{r}{2R} - \frac{5}{4}.$$

Proposed by Marin Chirciu-Romania

U.2159 If $x, y, z > 0$ and $\lambda \ge 0$ then:

$$\frac{1}{y(x+\lambda y)} + \frac{1}{z(y+\lambda z)} + \frac{1}{x(z+\lambda x)} \ge \frac{27}{(\lambda+1)(x+y+z)^2}.$$

Proposed by Marin Chirciu-Romania

U.2160 In ΔABC the following relationship holds: $2\left(3 - \frac{2r}{R}\right) \le \frac{a^2}{r_a^2} + \frac{b^2}{r_b^2} + \frac{c^2}{r_c^2} \le \frac{2R}{r}$.

Proposed by Marin Chirciu-Romania

U.2161 In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{\csc^4 A + (\csc B + \csc C)^4}{\csc B \csc C} \ge \frac{34R}{r}.$$

Proposed by Marin Chirciu-Romania

U.2162 In ΔABC the following relationship holds:

$$\sum_{cyc} \frac{IA}{2IA^3 + IB^3} \le \frac{2r}{R^3}.$$

Proposed by Marin Chirciu-Romania

U.2163 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{s_a^2}{s_c}\le\frac{9R}{2}\left(\frac{R}{2r}\right)^6.$$

Proposed by Marin Chirciu-Romania

U.2164 If $a,b,c>0, a+b+c=1$ and $0\le\lambda\le\frac{1}{4}$ then:

$$\frac{a^5}{b}+\frac{b^5}{c}+\frac{c^5}{a}+\lambda abc\ge\frac{\lambda+1}{27}.$$

Proposed by Marin Chirciu-Romania

U.2165 Solve for real numbers: $\begin{cases}\left(\frac{x^2+3n^2}{4n^2}\right)^4=\frac{2}{n}y-1\\ \left(\frac{y^2+3n^2}{4n^2}\right)^4=\frac{2}{n}z-1\\ \left(\frac{z^2+3n^2}{4n^2}\right)^4=\frac{2}{n}x-1\end{cases}$

Proposed by Marin Chirciu-Romania

U.2166 If $a,b>0$ then: $(3a+3b)^5+(4a)^5a\ge15(4a)^4b.$

Proposed by Marin Chirciu-Romania

U.2167 If $a,b,c>0$ then:

$$\sum_{cyc}\frac{2a+1}{\sqrt{4a^4+1}}+\sum_{cyc}\frac{2a^2+1}{2a+1}\ge6.$$

Proposed by Marin Chirciu-Romania

U.2168 If $x,y,z>0$ and $\lambda\ge1$ then:

$$\frac{x}{\lambda x+y+z}+\frac{y}{x+\lambda y+z}+\frac{z}{x+y+\lambda z}\le\frac{3}{\lambda+2}.$$

Proposed by Marin Chirciu-Romania

U.2169 If $a,b,c>0$ then:

$$\sum_{cyc}\frac{a}{\sqrt[3]{a^3+26abc}}\ge1.$$

Proposed by Marin Chirciu-Romania

U.2170 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{\sin^5\frac{A}{2}}{\sin^3\frac{B}{2}}\geq 1-\frac{r}{2R}.$$

Proposed by Marin Chirciu-Romania

U.2171 If $x,y,z>0, xy+yz+zx=1$ and $0\leq\lambda\leq\frac{3}{2}$ then:

$$\frac{1}{\lambda x^2+1}+\frac{1}{\lambda y^2+1}+\frac{1}{\lambda z^2+1}\leq\frac{9}{\lambda+3}.$$

Proposed by Marin Chirciu-Romania

U.2172 If $a,b,c>0, a^2+b^2+c^2=12$ and $n\in\mathbb{N}, n\geq 2$ then

$$\sum_{cyc}\frac{a^{2n}}{\sqrt{a^3+1}}\geq 4^n.$$

Proposed by Marin Chirciu-Romania

U.2173 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{\cos\frac{A}{2}}{h_a}\geq\frac{9}{2s}.$$

Proposed by Marin Chirciu-Romania

U.2174 In ΔABC the following relationship holds:

$$\frac{4R+r}{3\sqrt{3}R^2r}\leq\sum_{cyc}\frac{\cot\frac{A}{2}}{bc}\leq\frac{4R+r}{6\sqrt{3}Rr^2}.$$

Proposed by Marin Chirciu-Romania

U.2175 If $a,b,c,d,e>0, abcde=1$ then: $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a}\geq\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d}+\sqrt{e}.$

Proposed by Marin Chirciu-Romania

U.2176 If $a,b,c>0, abc=r^3$ and $n\in\mathbb{N}$ then:

$$\frac{a^{2n+1}}{b+c}+\frac{b^{2n+1}}{c+a}+\frac{c^{2n+1}}{a+b}\geq\frac{3}{2}r^{2n}.$$

Proposed by Marin Chirciu-Romania

U.2177 If $a,b,c>0, a^2+b^2+c^2=3$ and $0\leq\lambda\leq 1$ then:

$$(\lambda+1)(a^4+b^4+c^4)-\lambda(a^3+b^3+c^3)\geq 3\lambda abc.$$

Proposed by Marin Chirciu-Romania

U.2178 In ΔABC the following relationship holds: $\sum_{cyc}(s_b+s_c)(b^3+c^3)\geq 32\sqrt{3}F^2$.

Proposed by Marin Chirciu-Romania

U.2179 Let the integral:

$$f(a,b)=\int_{-1}^{1} a^{\frac{1}{2}x(x+1)}b^{\frac{1}{2}x(x-1)}\,dx$$

then prove this sharp-inequality:

$$2\leq f\left(\frac{a+b}{2},\sqrt{ab}\right)\leq\sqrt{5}$$

Proposed by Srinivasa Raghava-India

U.2180 If we have the integral relation:

$$\left(\int_{-\infty}^{\infty}\frac{\sin\left(x-\frac{1}{x}\right)\cos\left(x+\frac{1}{x}\right)}{\sqrt[3]{x}}dx\right)^2=\frac{3\pi\beta}{4}+\left(\int_{-\infty}^{\infty}\frac{\sin\left(x+\frac{1}{x}\right)\cos\left(x-\frac{1}{x}\right)}{\sqrt[3]{x}}dx\right)^2$$

then prove that $\beta^4+3\beta^2+9=0$.

Proposed by Srinivasa Raghava-India

U.2181 Find:

$$\Omega=\lim_{n\to\infty}\frac{(n-1)!\left(1+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}\right)}{n\Gamma\left(n+\frac{1}{2}\right)}$$

Proposed by Daniel Sitaru,Cătălin Pană-Romania

U.2182 Find:

$$\Omega=\lim_{n\to\infty}\frac{\log\left(e\cdot(n+1)^{\frac{n+1}{n}}\right)\cdot\log\left(e\cdot n^{\frac{n+2}{n+1}}\right)}{\log(e^n\cdot(n+1)n^n)\cdot\log(e\cdot(n+1)n^{n+1})}$$

Proposed by Daniel Sitaru,Ramona Nălbaru-Romania

U.2183 Find:

$$\Omega=\int\frac{x^3+x-\tan^{-1}x}{(x^2+1)\left(x^2-(\tan^{-1}x)\right)}dx\,,x>0$$

Proposed by Daniel Sitaru, Luiza Dumitrescu-Romania

U.2184 If $a,b,c>0,a+b+c=3$ then:

$$\sqrt{a+\sqrt{b+\sqrt{c}}}+\sqrt{b+\sqrt{c+\sqrt{a}}}+\sqrt{c+\sqrt{a+\sqrt{b}}}\le 3\sqrt{1+\sqrt{2}}$$

Proposed by Daniel Sitaru, Gigi Zaharia-Romania

U.2185 If $a,b,c,d>0, a+b+c+d=1$ then:

$$\sum_{cyc} a^2cd\cdot\cot^{-1}(b)\ge 4abcd\cdot\cot^{-1}\left(\frac{1}{4}\right)$$

Proposed by Daniel Sitaru, Elena Nedelcu-Romania

U.2185 If $0<a\le b$ then:

$$\int_a^b\int_a^b\left(\sqrt{x(2y+x)}+\sqrt{y(2x+y)}\right)dxdy\le\sqrt{3}(b-a)^2(a^2+ab+b^2)$$

Proposed by Daniel Sitaru, Ileana Duma-Romania

U.2186 Let $(a_n)_{n\ge1},(b_n)_{n\ge1},a_n,b_n\in\mathbb{R}_+^*=(0,\infty)$ such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{n\cdot a_n}=a$, $\lim\limits_{n\to\infty}\frac{b_{n+1}}{b_n\sqrt[n]{a_n}}=b$,

$a,b\in\mathbb{R}$. Find:

$$\Omega=\lim_{n\to\infty}\left(\sqrt[n+1]{b_{n+1}}-\sqrt[n]{b_n}\right)$$

Proposed by D.M. Bătinețu-Giurgiu, Daniel Sitaru-Romania

U.2187 Find:

$$\Omega=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{\sin 2x\cdot\cos 4x\cdot\cos 8x}{\sin^5\left(\frac{\pi}{4}-x\right)+\cos^5\left(\frac{\pi}{4}-x\right)}dx$$

Proposed by Daniel Sitaru, Camelia Dană-Romania

U.2188 Find:

$$\Omega=\lim_{n\to\infty}e^{5n+1}\cdot\tan^5 n\left(\int_0^n e^{5x}(\tan^4 x+\tan^5 x+\tan^6 x)dx\right)^{-1}$$

Proposed by Daniel Sitaru-Romania

U.2189 Find $a\in\mathbb{R}$ such that:

$$\sum_{n=0}^{\infty}\frac{2^nF_n}{\left(2021+\sqrt{4084445}\right)^n}=\frac{1}{a}$$

Proposed by Syed Shahabudeen-India

U.2190 For $n > 1$ prove that:

$$2\int_0^{\pi} \log(1-2\alpha\cos 2x+\alpha^2)\sin x\sum_{k=0}^{\infty}(-1)^k J_{2k+1}((2n-1)x)\,dx=\frac{\pi\alpha^n(\alpha(n-1)-n)}{2\alpha n(n-1)}$$

where J_{2k+1} is the Bessel function of first Kind.

Proposed by Tobi Joshua-Nigeria

U.2191 If $L(x)=\frac{6}{\pi^2}\left(Li_2(x)+\frac{1}{2}\log x\log(1-x)\right)$, find: $\Omega=\int_0^1 L(x)\cdot Li_2(x)\,dx$

Proposed by Togrul Ehmedov-Azerbaijan

U.2192 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{r_a}{(r_b+r_c)^2}\geq\frac{1}{2R}$$

Proposed by Marian Ursărescu-Romania

U.2193 Prove that: $\psi_1\left(\frac{1}{8}\right)+\psi_1\left(\frac{5}{8}\right)=32G+4\pi^2$

where $\psi_1(x)$ —is trigamma function and G is Catalan's constant.

Proposed by Vasile Mircea Popa-Romania

U.2194 Let $a,b,c>0, a+b+c=3$. Prove that:

$$2\sum_{cyc}\frac{a^2-a}{a+b^2+c}\geq\sum_{cyc}\frac{a^2-1}{a^2+b^2+c}$$

Proposed by Choy Fai Lam-Hong Kong

U.2195 Let $a,b,c>0, ab+bc+ca=3$. Prove that:

$$1\leq\frac{a}{b+c+bc}+\frac{b}{c+a+ca}+\frac{c}{a+b+ab}\leq\frac{a^2+b^2+c^2}{3}$$

Proposed by Choy Fai Lam-Hong Kong

U.2196 Find x:

$$\sqrt{2+\sqrt[3]{6+\sqrt[4]{2+\sqrt[3]{14+\sqrt{2+\sqrt[3]{6+\sqrt[4]{14+x}}}}}}}=\sqrt[4]{14+\sqrt[3]{6+\sqrt{2+\sqrt[4]{14+\sqrt[3]{6+\sqrt{2+x}}}}}}$$

Proposed by Tran Quoc Thinh-Vietnam

U.2197 If $x, y, z > 0$ and $k \in \mathbb{R}$ then solve the system:

$$\begin{cases} x^2 = k(1+x)^3 \\ y^2 = k(1+y)^3 \\ z^2 = k(1+z)^3 \\ xyz = 1 \end{cases}$$

Proposed by Nikos Ntorvas-Greece

U.2198 Prove the closed form:

$$\int_0^1 \frac{\log(1-x^2)}{\sqrt{x}(1+\sqrt{x})}dx = \frac{7}{2}\log^2 2 - \frac{5}{4}\zeta(2)$$

where $\zeta(s), \Re(s) > 1$ is the Euler-Riemann zeta function.

Proposed by Ankush Kumar Parcha-India

U.2199 In ΔABC the following relationship holds:

$$2\prod_{cyc}\frac{m_a}{r_a} \le 1 + \frac{1}{4}\sum_{cyc}\left(\frac{a}{w_a}\right)^2$$

Proposed by Bogdan Fuștei-Romania

U.2200 In $\Delta ABC, I$ –incenter, R_a, R_b, R_c –circumradii of $\Delta BIC, \Delta CIA, \Delta AIB$. Prove that:

$$\frac{R_a}{w_a} + \frac{R_b}{w_b} + \frac{R_c}{w_c} \ge 2\left(\frac{r_a + r_b + r_c + AI + BI + CI}{m_a + m_b + m_c + h_a + h_b + h_c - 3r}\right)^2$$

Proposed by Bogdan Fuștei-Romania

U.2201 In $\Delta ABC, n_a$ –Nagel's cevian, g_a –Gergonne's cevian, the following relationship holds:

$$b^2 + c^2 \ge 2(n_a g_a + r r_a) \ge 2bc$$

Proposed by Bogdan Fuștei-Romania

U.2202 Prove the summation:

$$\sum_{n=-\infty}^{\infty}\frac{(-1)^{\frac{1}{2}n(n+1)} + (-1)^n}{8n^3 + 4n^2 + 2n + 1} = \frac{\pi}{4}\left(1 + \frac{e^{\frac{\pi}{4}}\operatorname{sech}\left(\frac{\pi}{4}\right)}{e^{\frac{\pi}{4}} - 1}\right)$$

Proposed by Srinivasa Raghava-India

U.2203 For any complex number q with $\Re(q) \le 1$, let

$$R(q) = \int_0^q \frac{e^x - 1}{x}\log^2\left(\frac{q}{x}\right)dx$$

then prove the relation: $\int_0^\infty e^{-q}\left(1+\frac{1}{q}\right)R(q)dq = 2\big(\zeta(3)+\zeta(4)\big)$

Proposed by Srinivasa Raghava-India

U.2204 If we have the integrals:

$$\alpha = \int_{-\infty}^{\infty} e^{-\frac{\pi}{2}(x^2+x)}(-1)^{\frac{x}{2}(x+1)}\sqrt{1+\cosh\left(\frac{\pi x}{x}\right)}dx, \qquad \beta = \int_{-\infty}^{\infty} e^{-\frac{\pi}{2}(x^2+x)}(-1)^{x}\sqrt{1+\cosh\left(\frac{\pi x}{2}\right)}dx$$

then show that:

$$\left|\frac{\alpha}{\beta}\right| = \frac{e^{\frac{31\pi}{64}}\left(1+e^{\frac{\pi}{4}}\right)}{\sqrt[4]{2}\cdot\sqrt{1+e^{\frac{\pi}{2}}}}$$

Proposed by Srinivasa Raghava-India

U.2205 Prove that $a = -\cos\frac{8\pi}{31}\cos\frac{9\pi}{31}\cos\frac{14\pi}{31}$ is the smallest real root of the equation

$$32768x^5 - 36864x^4 + 10240x^3 - 320x^2 - 88x - 1 = 0$$

Proposed by Vasile Mircea Popa-Romania

U.2206 $A, B \in M_3(\mathbb{C}), AB = BA, \det A = 2, \det B = \frac{1+i\sqrt{7}}{2}, \det(A^2 - AB + B^2) = 0.$

Solve for real numbers: $\det(A - xB) = 0$. ***Proposed by Daniel Sitaru-Romania***

U.2207 If $a, b, c > 0$ then: $(3ab)^c \cdot (3bc)^a \cdot (3ca)^b \le (a^2+b^2+c^2)^{a+b+c}$

Proposed by Daniel Sitaru-Romania

U.2208 Find:

$$\Omega = \lim_{x\to\infty}\left(\log^2 x \cdot \int_0^1 \frac{t\cdot x^t}{(1+x^t)^2}dx\right)$$

Proposed by Daniel Sitaru-Romania

U.2209 In $\Delta ABC, n_a$ –Nagel's cevian, g_a –Gergonne's cevian, the following relationship holds:

$$2m_a \ge \sqrt{(n_a+g_a)^2 + (r_a - r)(2h_a - r_b - r_c)}$$

Proposed by Bogdan Fuștei-Romania

U.2210 If ω –Brocard's angle in ΔABC then:

$$\sqrt{\frac{2}{\sin\omega}}\cdot\sum_{cyc}\sqrt{\frac{m_a}{h_a}} \ge \sum_{cyc}\frac{m_b+m_c}{m_a}$$

Proposed by Bogdan Fuștei-Romania

U.2211 Let $I, I_a, I_b, I_c, \Delta DEF$ be the incenter, the excenters and the intouch triangle in ΔABC. Let r_1, r_2, r_3 —be inradii of $\Delta AEF, \Delta BDF, \Delta CDE$. Prove that:

$$r_1 + r_2 + r_3 = \frac{[I_a I_b I_c] - [ABC]}{2R\left(\cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}\right)}$$

Proposed by Mehmet Șahin-Turkiye

U.2212 If $X \in INt(\Delta ABC), XA' = XB' = XC' = R, R$ —circumradii,$XA' \perp BC, XB' \perp CA, XC' \perp AB, a', b', c', h_a', h_b', h_c'$ — sides and altitudes in $\Delta A'B'C'$, then:

$$\sum_{cyc} \frac{a'}{h_a} \cdot \left(\sum_{cyc} \frac{a}{h_a'}\right)^{-1} = [A'B'C']([ABC])^{-1}$$

Proposed by Mehmet Șahin-Turkiye

U.2213 Let ω — be the Brocard's angle of ΔABC.Prove that:

$$\sum_{cyc} \frac{m_a}{w_a} \cdot \sqrt[4]{\frac{m_a}{h_a}} \geq \sqrt[4]{\frac{\sin\omega}{8}} \cdot \sum_{cyc} \frac{b+c}{a}$$

Proposed by Bogdan Fuștei-Romania

U.2214 Let $a, b, c \geq 0, ab + bc + ca + 2abc \geq 1$. Find Min-value of P.

$$P = \sqrt{a+1} + \sqrt{b+1} + \sqrt{c+1}$$

Proposed by Phan Ngoc Chau-Vietnam

U.2215 For $a_1, a_2, \dots, a_n \geq 0$: $\sum_{1 \leq i < j \leq n} a_i a_j = 1$. Set $t = \sum_{i=1}^{n} a_i \geq \sqrt{\frac{2n}{n-1}}$ then:

$$\sum_{i=1}^{n} \frac{\sqrt{3a_i^2 - 3a_i t + 4}}{a_i^2 + a_i(1-t) + 1} \geq 2n - 2$$

When does equality holds?

Proposed by Phan Ngoc Chau-Vietnam

U.2216 In $\Delta ABC, n_a$ —Nagel's cevian, the following relationship holds:

$$\frac{n_a}{h_a} + \frac{n_b}{h_b} + \frac{n_c}{h_c} \geq 1 + \frac{2R}{r}\left(\sum_{cyc} \sin\frac{A}{2} - 1\right)$$

Proposed by Bogdan Fuștei-Romania

U.2217 In ΔABC the following relationship holds:

$$2(ab+bc+ca) \geq a^2+b^2+c^2+4F\sqrt{\sum_{cyc}\frac{a(s-a)}{b(s-b)}}$$

Proposed by Bogdan Fuștei-Romania

U.2218 In $\Delta ABC, AD$ −internal bisector, r_1, r_2 −inradii of $\Delta ABD, \Delta ACD$. Prove that:

$$c<b \Leftrightarrow r_a<r_2$$

Proposed by Mehmet Șahin-Turkiye

U.2219 In $\Delta ABC, n_a$ −Nagel's cevian, the following relationship holds: $\sum_{cyc}\frac{h_a}{s-n_a} \geq 3\sqrt{3}+\sum_{cyc}\frac{n_a}{r_a}$

Proposed by Bogdan Fuștei-Romania

U.2220 If $a,b,c>0$ then:

$$a+b+c+\frac{\sqrt{2}-1}{2}\sum_{cyc}|a-b| \leq \sum_{cyc}\sqrt{\frac{a^2+b^2}{2}}.$$

Proposed by Tran Hong-Vietnam

U.2221 If $x,y,z \geq 0, x+y+z=1, n \geq 2$ then:

$$(n+1)(xy+yz+zx) \leq n(x^2+y^2+z^2)+9xyz.$$

Proposed by Marin Chirciu-Romania

U.2222 If in $\Delta ABC, b^2+c^2=3a^2$ then: $\frac{2}{h_a}\sqrt{\frac{bc}{5}}+\frac{w_b}{h_b}+\frac{w_c}{h_c}<1+\frac{r}{R}.$

Proposed by Daniel Sitaru-Romania

U.2223 Find:

$$\Omega=\int_0^1\frac{\log(1-x)\cdot\log^2 x}{x}dx.$$

Proposed by Abdul Mukhtar-Nigeria

U.2224 Find:

$$\Omega=\int_0^1\left(\sum_{n=1}^{\infty}\left(\frac{d^n(x^n\log x)}{dx^n}\cdot\frac{x^n}{n!\cdot n^2}\right)\right)dx.$$

Proposed by Abdul Hafeez Ayinde-Nigeria

U.2225 Find:

$$\Omega=\int\frac{e^{x}\sin(e^{x})\log(sin^{-1}(cos(e^{x})))}{\sqrt{1-\cos(2e^{x})}}dx.$$

Proposed by Ahmad Hajjaj-Algerie

U.2226 Find:

$$\Omega=\int\frac{\log x}{\sqrt{\pi+2x}}dx\,,\ \ x>0.$$

Proposed by Ekpo Samuel-Nigeria

U.2227 Without the use of double integrals prove that:

$$L\left\{\operatorname{erf}\left(\frac{q\pi t}{p}\right)\right\}_{s}=\frac{q^{2}\pi^{2}e^{\frac{s^{2}p^{2}}{4q^{2}\pi^{2}}}\operatorname{erf}c\left(\frac{sp}{2q\pi}\right)}{sp^{2}}.$$

Proposed by Abdul Hafeez Ayinde-Nigeria

U.2228 In ΔABC the following relationship holds:

$$\sqrt{\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}}+\sqrt{\frac{a^{3}+b^{3}+c^{3}}{3abc}}\le\left(1+\frac{\sqrt{6}}{2}\right)\cdot\frac{R^{2}}{4r^{2}}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2229 If x,y,z are different positive real numbers and $\sqrt{x}+\sqrt{y}+\sqrt{z}=1$ then prove:

$$\frac{xy}{x-y}\log\left(\frac{x}{y}\right)+\frac{yz}{y-z}\log\left(\frac{y}{z}\right)+\frac{zx}{z-x}\log\left(\frac{z}{x}\right)\le\frac{1}{3}.$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2230 Prove that:

$$\int_{0}^{\frac{2}{n}}\log\left(\Gamma\left(x+\frac{1}{n}\right)\right)dx=$$

$$=\frac{1}{2}\left(\log(2\pi)+1-\frac{2}{n}\right)\cdot\frac{2}{n}-\frac{2}{n^{2}}\left(\frac{3}{n}-1\right)\log\Gamma\left(\frac{3}{n}\right)-\log\left(\frac{G}{3}\right)+\left(1-\frac{1}{n}\right)\log\Gamma\left(\frac{1}{n}\right)+\log G\left(\frac{1}{n}\right)$$

where $G(z)$ is Barnes G –function.

Proposed by Kaushik Mahanta-India

U.2231 For $m,n\in\mathbb{N},m>2,n>1$ prove that:

$$\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\cdots\sum_{k_n=0}^{\infty}\frac{1}{\prod_{\alpha=1}^{m}(k_1+k_2+\cdots+k_n+\alpha)}=\frac{\Gamma(m-2)}{\Gamma(m+n-3)\Gamma(m+n-2)}.$$

Proposed by Kaushik Mahanta-India

U.2232 I_a, I_b, I_c −incenters in $\Delta ABC, \varphi_a, \varphi_b, \varphi_c$ −circumradii of $\Delta BCI_a, \Delta CAI_a, \Delta ABI_c$. Prove that:

$$\frac{1}{\varphi_a w_a}+\frac{1}{\varphi_b w_b}+\frac{1}{\varphi_c w_c}=\frac{1}{Rr}.$$

Proposed by Mehmet Şahin-Turkiye

U.2233 a, b, c −sides in $\Delta ABC, \sqrt{6a^2-2b^2+3c^2}, \sqrt{6b^2-2c^2+3a^2}, \sqrt{6c^2-2a^2+3b^2}$ −sides in $\Delta A'B'C'$. Find:

$$\Omega=\frac{[A'B'C']}{[ABC]}.$$

Proposed by Mehmet Şahin-Turkiye

U.2234 In acute $\Delta ABC, X\in Int(\Delta ABC), XA'\perp BC, XB'\perp CA, XC'\perp AB, XA'=h_a, XB'=h_b,$

$XC'=h_c, R'$ −circumradii of $\Delta A'B'C'$. Prove that: $R'\ge\frac{12r^3}{R^2}$.

Proposed by Mehmet Şahin-Turkiye

U.2235 In $\Delta ABC, m(\sphericalangle BAC)=90°, AD\perp BC, D\in(BC), r_1, r_2, r$ −inradii of $\Delta ABD, \Delta ACD, \Delta ABC$. Prove that:

$$\frac{r-r_1}{r+r_1}+\frac{r-r_2}{r+r_2}\ge\frac{2h_a}{2s+h_a}.$$

Proposed by Mehmet Şahin-Turkiye

U.2236 In acute $\Delta ABC, r_A$ −radii of circle tangent simultaneous to BC in the middle of BC and circumcircle of ΔABC(internal tangent). If r_b, r_C −are similarly defined then:

$$R_A+R_B+R_C=R-\frac{r}{2}.$$

Proposed by Mehmet Şahin-Turkiye

U.2237 For $(u_n)_{n\ge1}$ such that $u_1=1, u_{n+1}=\frac{\sqrt{u_n}}{n}+\frac{(n-1)^2}{n^2}u_n, n\ge1$ prove that:

$$\lim_{n\to\infty}u_n=\frac{1}{4}.$$

Proposed by Minh Vu-Vietnam

U.2238 In $\Delta ABC, 2s=a+b+c$ prove that:

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+\frac{4R^2+4Rr+3r^2-s^2}{2r^2+Rr}\geq\frac{b}{a}+\frac{c}{b}+\frac{a}{c}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2239 In $\Delta ABC, x=s-a, y=s-b, z=s-c$. Prove that: $\frac{n_a-g_a}{2}<m_a\leq\frac{2x+y+z-\sqrt{yz}}{\sqrt{3}}$.

Proposed by Nguyen Van Canh-Vietnam

U.2240 In ΔABC the following relationship holds:

$$\frac{r_a+r}{r_a-r}+\frac{r_b+r}{r_b-r}+\frac{r_c+r}{r_c-r}\leq\frac{s_b+s_c}{h_a}+\frac{s_c+s_a}{h_b}+\frac{s_a+s_b}{h_c}.$$

Proposed by Bogdan Fuștei-Romania

U.2241 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{a^3}{\sin^2\frac{A}{2}}=16Rs(R+R),\sum_{cyc}\frac{a^3}{\cos^2\frac{A}{2}}=16Rs(R-r).$$

Proposed by Mehmet Șahin-Turkiye

U.2242 Prove that:

$$\frac{\sin\left(\frac{\pi}{36}\right)}{\sin\left(\frac{2\pi}{9}\right)}=\sqrt[3]{\frac{2+\tan^2\left(\frac{\pi}{9}\right)}{1+14\cos\left(\frac{\pi}{9}\right)}}\cdot\sqrt[3]{\frac{2+\tan^2\left(\frac{\pi}{18}\right)}{\sqrt{3}+14\cos\left(\frac{\pi}{18}\right)}}\cdot\sqrt[3]{\frac{2+\tan^2\left(\frac{\pi}{36}\right)}{\frac{\sqrt{2}+\sqrt{6}}{2}+14\cos\left(\frac{\pi}{36}\right)}}.$$

Proposed by Mohammed Bouras-Morocco

U.2243 In ΔABC the following relationship holds:

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{\sqrt{3}(w_a+w_b+w_c)}{R(h_a+h_b+h_c)}.$$

Proposed by Bogdan Fuștei-Romania

U.2244 In ΔABC the following relationship holds:

$$\left(\frac{m_a}{h_a}+\frac{m_b}{h_b}+\frac{m_c}{h_c}\right)^3\geq\frac{27r_ar_br_c}{w_aw_bw_c}.$$

Proposed by Adil Abdullayev-Azerbaijan

U.2245 Let $M\in Int(\Delta ABC)$ and $x=MA, y=MB, z=MC$, then:

$$\left(\frac{x^2}{a^2}+t^2\right)\left(\frac{y^2}{b^2}+t^2\right)\left(\frac{z^2}{c^2}+t^2\right)\geq\frac{9}{4}t^4$$

Proposed by D.M. Bătinețu-Giurgiu,Claudia Nănuți -Romania

U.2246 Let $a, b, c, d > 0$ then:

$$\sum_{cyc}\left(\prod_{cyc}(a^4+3a^2+2)\right) \geq \frac{45}{2}(ab+ac+ad+bc+bd+cd)$$

Proposed by D.M. Bătinețu-Giurgiu,Claudia Nănuți -Romania

U.2247 Let $n \in \mathbb{N}, n \geq 3$ and $t, a_k \in \mathbb{R}_+^*, (\forall)k = \overline{1,n}$, then:

$$\prod_{k=1}^{n}(a_k^2+t^2) \geq \frac{3^{n-2}}{2^{2n-4}} t^{2(n-1)}(a_1+a_2+\cdots+a_n^2$$

Proposed by D.M. Bătinețu-Giurgiu,Claudia Nănuți -Romania

U.2248 In ΔABC holds: $\left(\frac{1}{r_a^2}+r\right)\left(\frac{1}{r_b^2}+r\right)\left(\frac{1}{r_c^2}+r\right) \geq \frac{3}{4}$

Proposed by D.M. Bătinețu-Giurgiu,Claudia Nănuți -Romania

U.2249 Let $x, y > 0$, then in ΔABC holds:

$$(x^2+y^2)(r_a^2+r_b^2+r_c^2) \geq 6\sqrt{3}xy \cdot F + \sum_{cyc}(xr_a+yr_b)^2$$

Proposed by D.M. Bătinețu-Giurgiu,Claudia Nănuți -Romania

U.2250 Let $(a_n)_{n\geq 1}$ sequence of real numbers strictly positive such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n\sqrt[n]{(2n-1)!!}} = a > 0$ then find:

$$\Omega = \lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}} - \frac{n^2}{\sqrt[n]{a_n}}\right)$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase-Romania

U.2251 Let $t \geq 0$ and the sequences $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$ of real numbers strictly positive such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n \cdot n^{t+1}} = a > 0$ and $\lim\limits_{n\to\infty}\frac{b_{n+1}}{b_n \cdot n^t} = b > 0$, then find:

$$\Omega = \lim_{n\to\infty}\frac{1}{n}\sqrt[n]{\frac{a_n}{b_n}}$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase -Romania

U.2252 Let $M \in Int(\Delta ABC)$ and $x = MA, y = MB, z = MC$, then:

$$\left(\frac{x^2}{a^2}+t^2\right)\left(\frac{y^2}{b^2}+t^2\right)\left(\frac{z^2}{t^2}+t^2\right) \geq \frac{9}{4}t^4$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase -Romania

U.2253 Let $a, b, c, d > 0$, then:

$$\sum_{cyc}\left(\prod_{cyc}(a^4+3a^2+2)\right) \geq \frac{45}{2}(ab+ac+ad+bc+bd+cd)$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase -Romania

U.2254 Let $n \in \mathbb{N}, n \geq 3$ and $t, a_k \in \mathbb{R}_+^*, (\forall)k = \overline{1,n}$, then:

$$\prod_{k=1}^{n}\left(a_k^2+t^2\right) \geq \frac{3^{n-2}}{2^{2n-4}} t^{2(n-1)}(a_1+a_2+\cdots+a_n)^2$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase -Romania

U.2255 In ΔABC holds:

$$\left(\frac{1}{r_a^2}+r\right)\left(\frac{1}{r_b^2}+r\right)\left(\frac{1}{r_c^2}+r\right) \geq \frac{3}{4}$$

Proposed by D.M. Bătinețu-Giurgiu,Florică Anastase -Romania

U.2256 Let $x, y > 0$ then in ΔABC holds:

$$(x^2+y^2)\left(r_a^2+r_b^2+r_c^2\right) \geq 6\sqrt{3}xyF + \sum_{cyc}(xr_a-yr_b)^2$$

Proposed by D.M. Bătinețu-Giurgiu,Gheorghe Boroica-Romania

U.2257 Let $(a_n)_{n\geq 1}$ sequence of real numbers strictly positive such that

$\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n\sqrt[n]{(2n-1)!!}} = a > 0$. Find:

$$\Omega = \lim_{n\to\infty}\left(\frac{(n+1)^2}{\sqrt[n+1]{a_{n+1}}} - \frac{n^2}{\sqrt[n]{a_n}}\right)$$

Proposed by D.M. Bătinețu-Giurgiu,Nicolae Mușuroia-Romania

U.2258 Let $t \geq 0$ and $(a_n)_{n\geq 1}, (b_n)_{n\geq 1}$ sequence of real numbers such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n\cdot n^{t+1}} = a > 0$ and $\lim\limits_{n\to\infty}\frac{b_{n+1}}{b_n\cdot n^t} = b > 0$, then find:

$$\Omega = \lim_{n\to\infty}\left(\sqrt[n+1]{\frac{a_{n+1}}{b_{n+1}}} - \sqrt[n]{\frac{a_n}{b_n}}\right)$$

Proposed by D.M. Bătinețu-Giurgiu,Flaviu Cristian Verde-Romania

U.2259 Let $(x_n)_{n\geq 1}, x_n = \sum_{k=1}^{n} \frac{1}{k}$ and $(a_n)_{n\geq 1}$ sequence of real numbers strictly positive such that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n \cdot e^{x_n}} = a > 0$, then find: $\Omega = \lim_{n\to\infty} \left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$.

Proposed by D.M. Bătinețu-Giurgiu,Daniel Sitaru-Romania

U.2260 Let $x, y, z, t > 0$ then in ΔABC holds:

$$\left(\frac{x+y}{z} \cdot a^2 + t\right)\left(\frac{y+z}{x} \cdot b^2 + t\right)\left(\frac{z+x}{y} \cdot c^2 + t\right) \geq 18\sqrt{3} \cdot F$$

Proposed by D.M. Bătinețu-Giurgiu,Daniel Sitaru -Romania

U.2261 Let $t > 0$ then in $\Delta ABC, \mu(\hat{A}) = 90°$ holds: $4(b^4 + t)(c^4 + 1) \geq 3t(a^4 + t)$

Proposed by D.M. Bătinețu-Giurgiu,Daniel Sitaru -Romania

U.2262 Let $a, b, m, n > 0$, then: $(m^2a^2 + t)(m^2b^2 + t) \geq \frac{3}{4}t(m^2(a+b)^2 + t)$

Proposed by D.M. Bătinețu-Giurgiu,Daniel Sitaru -Romania

U.2263 Let $t \geq$ and $M \in Int(\Delta ABC)$ such that $x = MA, y = MB, z = MC$, then:

$$(x^2a^4 + t^2)(y^2b^4 + t^2)(z^2c^4 + t^2) \geq 12t^4(xy + yz + zx) \cdot F^2$$

Proposed by D.M. Bătinețu-Giurgiu,Daniel Sitaru -Romania

U.2264 Let $t > 0$ then in ΔABC holds:

$$(a^2 + b^2 + 2t^2)(b^2 + c^2 + 2t^2)(c^2 + a^2 + 2t^2) \geq 72t^4\sqrt{3} \cdot F$$

Proposed by D.M. Bătinețu-Giurgiu, Dan Nănuți-Romania

U.2265 Let $x, y, z > 0$ then in ΔABC holds:

$$\left(\frac{x^2a^2}{(y+z)^2} + 1\right)\left(\frac{y^2b^2}{(z+x)^2} + 1\right)\left(\frac{z^2c^2}{(x+y)^2} + 1\right) \geq \frac{27\sqrt{3}}{4} \cdot F$$

Proposed by D.M. Bătinețu-Giurgiu, Dan Nănuți -Romania

U.2266 In $\Delta ABC, n_a$ —Nagel's cevian, g_a —Gergonne's cevian and s_c —simmedians from C, holds:

$$(n_a^2 + 2)(g_b^2 + 2)(s_c^2 + 2) \geq 36 \cdot \frac{F^2}{R^2}$$

Proposed by D.M. Bătinețu-Giurgiu, Dan Nănuți -Romania

U.2267 Let $(a_n)_{n\geq 1}$ sequence of real numbers strictly positive such that

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n \sqrt[n]{(2n-1)!!}} = a > 0$. Find: $\Omega = \lim_{n\to\infty} \left(\sqrt[n+1]{a_{n+1}} - \sqrt[n]{a_n}\right)$

Proposed by D.M. Bătinețu-Giurgiu, Dan Nănuți -Romania

U.2268 Let $(a_n)_{n\geq 1}$ sequence of real numbers strictly positive such that $\lim\limits_{n\to\infty}\frac{a_{n+1}}{a_n(\sqrt[n]{n!})^2}=a>0$. Find:

$$\Omega=\lim_{n\to\infty}\left(\frac{(n+1)^3}{\sqrt[n+1]{a_{n+1}}}-\frac{n^2}{\sqrt[n]{a_n}}\right)$$

Proposed by D.M. Bătinețu-Giurgiu,Neculai Stanciu-Romania

U.2269 Find the values of a,b and k such that:

$$\lim_{x\to 0}\frac{2^{\sin 2x}+3^{\cos 3x}+4^{\sin 4x}+5^{\cos 5x}+a}{bx^k}=10.$$

Proposed by Jalil Hajimir-Canada

U.2270"Prove or disprove "in any ΔABC holds:

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+\frac{R^3}{r^3}\geq 8+\frac{a^2}{c^2}+\frac{c^2}{b^2}+\frac{b^2}{a^2}"$$

Proposed by Nguyen Van Canh-Vietnam

U.2271 Let $a,b,c>0$. Prove that:

$$\sum_{cyc}\frac{2b+5c}{\sqrt{a^2+3bc}}\geq\frac{21}{2}.$$

Proposed by Tran Quoc Thinh-Vietnam

U.2272 If $t\geq 1$ then prove that in any triangle ABC with usual notations holds:

$$\sum_{cyc}\frac{1}{(\sin B+\sin C)^t}+\frac{3^{t+1}R^t}{2^t s^t}\geq 4\sum_{cyc}\frac{1}{(\sin A+\sin B)^t+(\sin A+\sin C)^t}.$$

Proposed by Marius Drăgan, Neculai Stanciu-Romania

U.2273 Prove that ABC is isosceles right triangle if and only if $F=\frac{(a+b)\sqrt{ab}}{4}$, (where we denote by a,b the legs and F the area.

Proposed by Neculai Stanciu-Romania

U.2274 Solve for real numbers:

$$[x]+\frac{1}{\left[\frac{1}{1-x}\right]}=1,[*]-\text{Gif}.$$

Proposed by Neculai Stanciu-Romania

U.2275 Determine the general terms of $(x_n)_{n\geq 0}$, $(y_n)_{n\geq 0}$, which are given by $\begin{cases}x_{n+1}=-x_n+3y_n\\ y_{n+1}=-3x_n+5y_n\end{cases}$, with $n \geq 0, x_0 = 2, y_0 = 1$.

Proposed by Neculai Stanciu-Romania

U.2276 Let $a, b, c > 0$. Prove that:

$$\sum_{cyc} \sqrt{\frac{b+c}{a^2+3bc}} \geq 3\sqrt{\frac{3}{2(a+b+c)}}.$$

Proposed by Tran Quoc Thinh-Vietnam

U.2277 Determine the set: $A = \left\{(a,b,c) \in \mathbb{N}^* \times \mathbb{N}^* \times \mathbb{N}^* \middle| \frac{5ab-1}{abc+1} \in \mathbb{N}\right\}$.

Proposed by Neculai Stanciu-Romania

U.2278 Let $a, b > 0$ and $x \in \left(0, \frac{\pi}{2}\right)$. Prove that:

$$(a+b) \cdot \frac{\sin x}{x} + \frac{2ab}{a+b} \cdot \frac{\tan x}{x} > \frac{6ab}{a+b}$$

$$a^2 \tan^k x + b^2 \sin^k y > 2abx^k, k > 0.$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

U.2279 Find:

$$\Omega = \int_1^{2022} \tan^{-1}\left(\frac{2023 - x - \tan^{-1}(2023 - x)}{x - \tan^{-1} x}\right) dx.$$

Proposed by Neculai Stanciu-Romania

U.2280 If ABC is a triangle with usual notations prove that:

$$\frac{w_a + w_b}{w_c a^4} + \frac{w_b + w_c}{w_a b^4} + \frac{w_c + w_a}{w_b c^4} \geq \frac{2}{3R^4}.$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

U.2281 Find:

$$\Omega = \int_0^1 \frac{\frac{\pi}{4} - \tan^{-1} x}{1 - x} dx.$$

Proposed by Asmat Qatea-Afghanistan

U.2282 Prove that the equation $a^x = bx^2 + cx + d$ it has at most three real roots for all $a > 1$,

$b \in \mathbb{R}_+, c, d \in \mathbb{R}$.

Proposed by Neculai Stanciu-Romania

U.2283 Let ω be the Brocard's angle in ΔABC. Prove that:

$$4+\frac{2}{\sin^2\omega}>\sum_{cyc}(n_a+g_a)\sqrt{\frac{r_b+r_c}{h_a^3}}+\sum_{cyc}\frac{n_a g_a}{h_a^2}\left(2-\frac{9\sqrt{2r_br_c}}{n_a+g_a+\sqrt{2r_br_c}}\right).$$

Proposed by Bogdan Fuștei-Romania

U.2284 Determine all real numbers x which satisfy the equation: $\frac{1}{x-3\sqrt{x}+2}+\frac{1}{x-\sqrt{x}}=\frac{2}{x-2\sqrt{x}}$.

Proposed by Neculai Stanciu-Romania

U.2285 For $x,y,z>0$ prove that:

$$\frac{x^3}{x^2+y^2}+\frac{y^3}{y^2+z^2}+\frac{z^3}{z^2+x^2}\geq\frac{x+y+z}{2}.$$

Proposed by Neculai Stanciu-Romania

U.2286 Find:

$$\Omega=\int_0^\infty\int_0^\infty\int_0^\infty\frac{\cos x\sin(x+y)\cos y\sin(y+z)\cos z\sin(z+x)}{xyz(x+y)(y+z)(z+x)}dxdydz.$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2287 If $X\in Int(\Delta ABC), XA'=XB'=XC'=R, R$ −circumradii, $XA'\perp BC, XB'\perp CA, XC'\perp AB$, I_a −excenters then $4[A'B'C']=[I_aI_bI_c]$.

Proposed by Mehmet Șahin-Turkyie

U.2288 If in triangle ABC we have $R=\frac{c\sqrt{ab}}{a+b}$, then compute r in terms of a.

Proposed by Neculai Stanciu-Romania

U.2289 Find:

$$\Omega=\lim_{x\to\infty}\left(x\Gamma\left(\frac{2x}{4x+1}\right)-x\sqrt{\pi}\right).$$

Proposed by Jalil Hajimir-Canada

U.2290 Find:

$$\Omega=\lim_{x\to 0}\frac{\sin x-\sin^{-1}x}{\sinh x-\sinh^{-1}x}.$$

Proposed by Jalil Hajimir-Canada

U.2291 Prove:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin^3(x + \sin 3x + \cos 3x)dx < \frac{\pi}{4}.$$

Proposed by Jalil Hajimir-Canada

U.2292 Solve: $\sinh^{-1} x \geq \tanh^{-1} x$.

Proposed by Jalil Hajimir-Canada

U.2293 Prove:

$$\int_0^1 \frac{x \tan^{-1} x}{1 + \tan^{-1} x} dx < \frac{1}{4}.$$

Proposed by Jalil Hajimir-Canada

U.2294 Prove that:

$$\lim_{n\to\infty} \frac{\prod_{m=1}^{n}(n^6 + m^6)}{2^n\left(2 + \sqrt{3}\right)^{\sqrt{3}n} n^{6n} e^{(\pi-6)n}} = \sqrt{2}.$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2295 Prove that:

$$\int_0^{\frac{\pi}{2}}\int_0^1 \tan^{-1}\left(\frac{\sin x}{u + \cos x}\right) dudx = \frac{\pi^2}{16} + \frac{3}{2}\log 2 - \frac{\pi}{4}.$$

Proposed by Asmat Qatea-Afghanistan

U.2296 Solve:

$$\log_x(x + 1) > \log_{[x]}([x] + 1), [*] - \text{GIF}.$$

Proposed by Jalil Hajimir-Canada

U.2297 Let n be a positive integer. Prove: $n^{n+1} \leq (n + 1)^n \sqrt[n]{n!}$.

Proposed by Jalil Hajimir-Canada

U.2298 Solve: $x^{[x]} + [x]^x = 3, [*] - \text{GIF}$.

Proposed by Jalil Hajimir-Canada

U.2299 Solve:

$$x\left[\sin\frac{\pi x}{2}\right] = [x]\sin\frac{\pi x}{2}, [*] - \text{GIF}.$$

Proposed by Jalil Hajimir-Canada

U.2300 Solve:

$$\begin{cases} x^2+y^2+z^2-2x-4y-6z=2 \\ 4x+3y+2z=16 \end{cases}.$$

Proposed by Jalil Hajimir-Canada

U.2301$ABCDA'B'C'D'$ −right parallelepiped, M,N,P,Q −midlle of $(AB),(BC),(A'D'),(C'D')$.

If $Volume[ABCDA'B'C'D'] \geq 1$ then $Area[MNPQ] \geq \frac{\sqrt{3}}{2}$.

Proposed by Radu Diaconu-Romania

U.2302 In ΔABC the following relationship holds:

$$\frac{\hat{A}\cdot r_a\cdot OI_a^2}{m_a}+\frac{\hat{B}\cdot r_b\cdot OI_b^2}{m_b}+\frac{\hat{C}\cdot r_c\cdot OI_c^2}{m_c}\geq\frac{32\pi r^3}{R}.$$

Proposed by Radu Diaconu-Romania

U.2303 In acute $\Delta ABC, H$ −orthocenter. Prove that:

$$\frac{18r^4}{s^2R^2}\leq\frac{(AH+BH+CH)w_aw_bw_c}{(s_a+s_b+s_c)h_ah_bh_c}\leq\frac{1}{6}.$$

Proposed by Radu Diaconu-Romania

U.2304 If in $\Delta ABC, \mu(\hat{A})=90°$, then:

$$\frac{1}{r_b}+\frac{1}{r_c}+\frac{r+a}{F}+\frac{\tan^2\frac{B}{2}}{a-c}>\frac{2}{R}+\frac{s}{R^2}+\frac{1}{\sqrt{2(a^2+c^2)}}.$$

Proposed by Radu Diaconu-Romania

U.2305 Find:

$$\Omega=\int_0^1\int_0^1\int_0^1\frac{dxdydz}{\sqrt{xyz}(1+4x)(1+3x+2y+z)}.$$

Proposed by Hikmat Mammadov-Azerbaijan

U.2306 Let $a,b,c>0, ab+bc+ca=3$. Prove that:

$$\frac{\sqrt{4-ab}+\sqrt{ab}}{2c+\sqrt{ab}}+\frac{\sqrt{4-bc}+\sqrt{bc}}{2a+\sqrt{bc}}+\frac{\sqrt{4-ca}+\sqrt{ca}}{2b+\sqrt{ca}}\geq\sqrt{3}+1.$$

Proposed by Phan Ngoc Chau-Vietnam

U.2307 Let $a,b,c>0$. Prove that:

$$\frac{1}{a+\sqrt{bc}}+\frac{1}{b+\sqrt{ca}}+\frac{1}{c+\sqrt{ab}}+\frac{3}{2\left(a+b+c-\sqrt[3]{abc}\right)}\geq\frac{27}{4(a+b+c)}.$$

Proposed by Phan Ngoc Chau-Vietnam

U.2309 If $u,v,w\in\mathbb{R}_+^*$, then prove that in any tetrahedron $[ABCD]$ holds:

$$\sum_{cyc}\frac{r_br_c}{r_a(ur_br_c+vr_ar_c+wr_ar_b)}\geq\frac{2}{(u+v+w)r}.$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

U.2310 Prove that if a,b,c are positive natural numbers with $\gcd(ab,c)=1$, then there exist an infinity of triplets (x,y,z) of natural numbers which satisfy $x^a+y^b=z^c$.

Proposed by Neculai Stanciu-Romania

U.2311 If $x,y\in\mathbb{R}_+^*, m\in\mathbb{R}_+$ then in any triangle ABC holds:

$$\frac{r_a^{m+2}}{(xr_a+yr_b)^m}+\frac{r_b^{m+2}}{(xr_b+yr_c)^m}+\frac{r_c^{m+2}}{(xr_c+yr_a)^m}\geq\frac{(4R+r)^2-2s^2}{(x+y)^2}.$$

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

U.2312 $\Omega_n=(f\circ f\circ\ldots\circ f)(x), f(x)=\frac{1}{\sqrt[2021]{1-x^{2021}}}$. Solve for real numbers:

$$\Omega_{2022}(x)=2021.$$

Proposed by Neculai Stanciu-Romania

U.2313 Find:

$$\Omega=\int_0^\infty\left(e^x\log(1-e^{-x})+1\right)^2dx.$$

Proposed by Jalil Hajimir-Canada

U.2314 Find:

$$\Omega=\lim_{x\to\frac{\pi}{6}}\frac{2\log\left(\Gamma(\sin x)\right)-\log\pi}{\Gamma(\sec 2x)-1}.$$

Proposed by Jalil Hajimir-Canada

U.2315 Solve:

$$\left[\frac{x}{50}\right]+\left[\frac{x}{20}\right]+\left[\frac{x}{10}\right]=343, [*]-\text{GIF}.$$

Proposed by Jalil Hajimir-Canada

U.2316 Evaluate:

$$\Omega = \lim_{x\to\infty} \frac{5^{\frac{1}{x^2}} - \cos\frac{1}{x}}{\frac{1}{x}\left(\log\left(1+\frac{2}{x}\right) - \sin\frac{3}{x}\right)}.$$

Proposed by Jalil Hajimir-Canada

U.2317 Solve: $|5^{\tan x} - 5^{1-\tan x}| < 4$.

Proposed by Jalil Hajimir-Canada

U.2318 Let $a,b,c \geq 0, ab+bc+ca = 1$. Find Min value of P:

$$P = \frac{ab+2c\sqrt{ab}+1}{c+\sqrt{ab}} + \frac{bc+2a\sqrt{bc}+1}{a+\sqrt{bc}} + \frac{ca+2b\sqrt{ca}+1}{b+\sqrt{ca}}.$$

Proposed by Phan Ngoc Chau-Vietnam

U.2319 Let $a,b,c \geq 0, ab+bc+ca > 0$. Prove that:

$$\frac{1}{(a+2\sqrt{bc})(2a+\sqrt{bc})} + \frac{1}{(b+2\sqrt{ca})(2b+\sqrt{ca})} + \frac{1}{(c+2\sqrt{ab})(2c+\sqrt{ab})} \geq \frac{1}{ab+bc+ca}.$$

Proposed by Phan Ngoc Chau-Vietnam

U.2320 Prove that:

$$\sum_{k=0}^{\infty} a_k 3^{-2k} = \frac{\sqrt{3}}{2}\left[\log\left(\sqrt{\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^3}\right) + \pi\right], \text{where } a_k = \sum_{j=0}^{k} \frac{(4k+2)(-1)^j}{(2j+1)(2k+1-2j)}$$

Proposed by Vincenzo Dima-Italy

U.2321 Find:

$$\Omega = \sum_{k=1}^{\infty} \frac{(4k)^3+3}{(4k)!}.$$

Proposed by Asmat Qatea-Afghanistan

U.2322 In ΔABC the following relationship holds:

$$(g_a^m + g_b^m + g_c^m)\left(\frac{\hat{A}^n}{g_a^m} + \frac{\hat{B}^n}{g_b^n} + \frac{\hat{C}^n}{g_c^m}\right)\left(\sum_{cyc} \cos^{2p}\frac{\hat{B}-\hat{C}}{2}\right) \geq \frac{27\pi^n\left(\frac{6r}{R}\right)^s}{s^{n+s}}.$$

Proposed by Radu Diaconu-Romania

U.2323 In ΔABC the following relationship holds:

$$\sum_{cyc}\left(\frac{A}{B}+\frac{B}{A}+A^2a\cdot\sin^2\frac{A}{2}+\pi\right)>\frac{\pi^2 s}{18}+2\sqrt{\pi}\cdot\sum_{cyc}\sqrt{\frac{\pi-A}{A}}.$$

Proposed by Radu Diaconu-Romania

U.2324 $VABC$ −tetrahedron, s_a, s_b, s_c −symedians from A, B, C, R −circumradii of ΔABC,

$\frac{s_a}{s}=\frac{s_b}{b}=\frac{s_c}{c}$. Prove that:

$$\frac{2([VAB]+[VBC]+[VCA]+[ABC])}{AB+BC+CA}\le\frac{R}{2}+\frac{s_a^2+s_b^2+s_c^2}{s_a+s_b+s_c}.$$

Proposed by Radu Diaconu-Romania

U.2325 If H −orthocenter in acute ΔABC then:

$$\max\left\{1+\frac{s}{AH},1+\frac{s}{BH},1+\frac{s}{CH}\right\}\ge\frac{12r(2r+s)}{R(7R-2r)}.$$

Proposed by Radu Diaconu-Romania

U.2326 In ΔABC the following relationship holds:

$$\max\left\{\frac{h_a}{A},\frac{g_b}{B},\frac{g_c}{C},\frac{s_a}{s},\frac{m_b}{b},\frac{n_c}{c}\right\}\ge\frac{9r}{2}\left(\frac{1}{\pi}+\frac{1}{3\sqrt{3}R}\right).$$

Proposed by Radu Diaconu-Romania

U.2327 In ΔABC the following relationship holds:

$$\frac{2}{3R^2}\cdot\min\{A,B,C\}\le\frac{A}{m_a^2+m_b^2}+\frac{B}{m_b^2+m_c^2}+\frac{C}{m_a^2+m_b^2}\le\frac{3\pi R^2}{8F^2}.$$

Proposed by Radu Diaconu-Romania

U.2328 In ΔABC the following relationship holds:

$$\left(\sqrt{Ar_ag_a}+\sqrt{Br_bg_b}+\sqrt{Cr_cg_c}\right)^2\le\pi\cdot(4R+r)(R+r).$$

Proposed by Radu Diaconu-Romania

U.2329 Find:

$$\Omega=\sum_{k=1}^{\infty}\frac{H_k2^{-k}}{k+1}.$$

Proposed by Vincenzo Dima-Italy

U.2330 In ΔABC the following relationship holds:

$$\left(\frac{h_a}{r_a}+\frac{h_b}{r_b}+\frac{h_c}{r_c}\right)^2+\frac{6r}{R}\geq 12$$

Proposed by Rahim Shahbazov-Azerbaijan

U.2331 If $\alpha,\beta\geq 0,\alpha^2+\beta^2\neq 0$ then in ΔABC holds:

$$\frac{n_a^4}{\alpha n_b^2+\beta n_c^2}+\frac{n_b^2}{\alpha n_c^2+\beta n_a^2}+\frac{n_c^4}{\alpha n_a^2+\beta n_b^2}\geq\frac{R^2}{\alpha+\beta}\sum_{cyc}\sin^2(B-C).$$

Proposed by Radu Diaconu-Romania

U.2332 In ΔABC the following relationship holds:

$$\frac{R}{2r}\sum_{cyc}\frac{m_a+m_b}{m_b+m_c}\geq\sum_{cyc}\frac{m_b+m_c}{m_a+m_b}$$

Proposed by Nguyen Van Canh-Vietnam

U.2333 In acute ΔABC holds:

$$\sum_{cyc}\frac{\cos A}{\cos B}+8\cos A\cos B\cos C\geq 4.$$

Proposed by Rahim Shahbazov-Azerbaijan

U.2334 $S=\sum_{k=1000}^{2020}\sqrt[2019]{k}\,,T=\sum_{k=1001}^{2020}\sqrt[2019]{k}\,,I=\int_{1000}^{2021}\sqrt[2019]{x}\,dx$

Prove that: $\frac{S+T}{2}<I<T$.

Proposed by Rajeev Rastogi-India

U.2335 In ΔABC the following relationship holds:

$$\sum_{cyc}n_an_b\geq\sum_{cyc}m_as_a$$

Proposed by Soumava Chakraborty-India

U.2336 Let $s,t\in\mathbb{R},\{L_n(s,t)\}_{n\geq 2}$ be a sequence given by

$L_n(s,t)=(n+1)^s\cdot\sqrt[n+1]{\left((n+1)!\right)^t}-n^s\cdot\sqrt[n]{(n!)^t}$ and let $f_{s,t}:\mathbb{R}_+^*\to\mathbb{R}_+^*$ be the function defined by $f_{s,t}(x)=(x+1)^s\left(\Gamma(x+2)\right)^{\frac{t}{x+1}}-x^s\left(\Gamma(x+1)\right)^{\frac{t}{x}}$, where Γ is the Gamma function. Find the following limits $\lim_{n\to\infty}L_n(s,t)$, respectively $\lim_{x\to\infty}f_{s,t}(x)$.

Proposed by D.M. Bătinețu-Giurgiu, Neculai Stanciu-Romania

U.2337 If x and y are positive natural numbers and p is a prime number such that

$\gcd(x^2 - x + 1, x + 1) = 1$, then solve the equation $x^4 + x - py^2 = 0$.

Proposed by Neculai Stanciu-Romania

U.2338 Prove that:

$$\frac{(x^2 + y^2 + z^2 - xy - yz - zx)^3}{(x - y)^2(y - z)^2(z - x)^2} \geq 27, \forall x, y, z \neq 0.$$

Proposed by Neculai Stanciu-Romania

U.2339 If in triangle ABC we consider $M \in (AB), N \in (AC), P \in (BC)$ such that MN is antiparallel to BC and $NP \parallel AB$, then determine the maximum of area of triangle MNP.

Proposed by Neculai Stanciu-Romania

U.2340 Prove that:

$$\sum_{cyc} \frac{z}{x + y + 6z} \leq \frac{3}{8}, \forall x, y, z > 0.$$

Proposed by Neculai Stanciu-Romania

U.2341 Solve: $\left[\frac{x}{2}\right]\left[\frac{x}{3}\right]\left[\frac{x}{4}\right] = \frac{x^3}{[x]}, [*] - \text{GIF}.$

Proposed by Jalil Hajimir-Canada

U.2342 Find:

$$\Omega = \int_0^{\frac{\pi}{4}} \frac{\cos^2 x}{\cos\left(x - \frac{\pi}{4}\right)} dx.$$

Proposed by Jalil Hajimir-Canada

U.2343 Find:

$$\Omega = \int_0^1 \frac{dx}{\sqrt[3]{1 - x^3}} dx.$$

Proposed by Jalil Hajimir-Canada

U.2344 Prove:

$$\sum_{k=1}^{n} \frac{k^2}{4^k} < 1.$$

Proposed by Jalil Hajimir-Canada

U.2345 Let a, b and c be positive real numbers. Prove:

$$\frac{a}{5a+4b}+\frac{b}{5b+4c}+\frac{c}{5c+4a}\le\frac{1}{3}.$$

Proposed by Jalil Hajimir-Canada

U.2346 Find:

$$\Omega=\int_{\frac{1}{3}}^{3}\frac{x+\sin\left(x^2-\frac{1}{x^2}\right)}{x\left(2+\cos\left(x+\frac{1}{x}\right)\right)}dx.$$

Proposed by Jalil Hajimir-Canada

U.2347 If $q\in\mathbb{N}, q\ge 1$ then in ΔABC the following relationship holds:

$$\left(\frac{n_a+g_a}{\mu(A)}\right)^q+\left(\frac{n_b+g_b}{\mu(B)}\right)^q+\left(\frac{n_c+g_c}{\mu(C)}\right)^q\ge 3\left(\frac{a^2+b^2+c^2}{\pi R}\right)^q.$$

Proposed by Radu Diaconu-Romania

U.2348 In ΔABC the following relationship holds:

$$\left(\sum_{cyc}\mu(A)\cdot g_a^2\right)\left(\prod_{cyc}\mu(A)\cdot g_a^2\right)\le\left(\frac{\pi s}{3}\right)^4\cdot F^2.$$

Proposed by Radu Diaconu-Romania

U.2349 In ΔABC prove that:

$$\frac{R}{2r}\sum_{cyc}w_a^2\ge\sum_{cyc}r_b r_c.$$

Proposed by Nguyen Van Canh-Vietnam

U.2350 If in $\Delta ABC, \mu(A)=90^\circ, D\in(AI), I$ –incenter, then:

$$\frac{8\sqrt{2}\cdot R^4}{r}\le\frac{AB^4}{DI}+\frac{CA^4}{DA}<32R^4\cdot\max\left\{\frac{1}{DI},\frac{1}{DA}\right\}.$$

Proposed by Radu Diaconu-Romania

U.2351 In ΔABC the following relationship holds:

$$27r^2\sqrt{\min\{A,B,C\}}\le\sqrt{A}\cdot w_b w_c+\sqrt{B}\cdot w_c w_a+\sqrt{C}\cdot w_a w_c\le\sqrt{\frac{\pi}{3}}\cdot s^2.$$

Proposed by Radu Diaconu-Romania

U.2352 In $\Delta ABC, c_a$ –cevian, holds:

$$\sum_{cyc}\frac{\sqrt[4]{ab}\cdot\sqrt{a+b}}{\sqrt{c_a}}\le R\sqrt{\frac{18}{r}}.$$

Proposed by Radu Diaconu-Romania

U.2354 In ΔABC the following relationship holds:

$$\min\left\{\sqrt{\frac{s-a}{s+a}\mu(A)},\sqrt{\frac{s-b}{s+b}\mu(B)},\sqrt{\frac{s-c}{s+c}\mu(C)}\right\}\le\sqrt{\frac{\pi}{9}\cdot\frac{4s^2-r^2-16Rr}{4s^2+r^2+8Rr}}.$$

Proposed by Radu Diaconu-Romania

U.2355 In ΔABC the following relationship holds:

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}\ge\frac{a+b+c}{2}+\frac{r^2(R-2r)}{3(R^2+r^2)}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2356 For all $-1\le x\le 1$ such that $|ax|\sqrt{1-x^2}\le 1$, prove that: $|a|\le\frac{x^2+1}{4}+\frac{4}{x^2+1}$.

Proposed by Nguyen Van Canh-Vietnam

U.2357 For all $x\in[-1,1]$ such tat $|ax+b|\sqrt{1-x^2}\le$, prove that: $|a|\le x^2-2x+3$.

Proposed by Nguyen Van Canh-Vietnam

U.2358 Let $a,b,c,d\in\mathbb{R}$ such that $|ax^3+bx^2+cx+d|\sqrt{1-d^2}\le 1,\forall|x|\le 1$. Prove that:

$$|a+b+c+d|\le 4.$$

Proposed by Nguyen Van Canh-Vietnam

U.2359 Let $f(x)=ax^{2n}+bx+c,a,b,c\in\mathbb{R},n\in\mathbb{N},n\ge 1,f(0),f(-1),f(1)\in[-1,1]$. Prove that:

$$|f(x)|\le\frac{2n-1}{\sqrt[2n-1]{4^n n^{2n}}},\forall|x|\le 1.$$

Proposed by Nguyen Van Canh-Vietnam

U.2360 Let $\alpha\ge 2$. In any ΔABC the following relationship holds: $\left(\frac{R}{2r}\right)^{\alpha}\sum_{cyc}h_bh_c\ge\sum_{cyc}r_br_c$.

Proposed by Nguyen Van Canh-Vietnam

U.2361 In ΔABC the following relationship holds:

$$\sum_{cyc}\tan\frac{B}{2}\tan\frac{C}{2}=\sum_{cyc}\frac{h_a}{2r_a+h_a}=\sum_{cyc}\frac{h_a-2r}{h_a}.$$

$$\sum_{cyc}\frac{b-c}{a}\cos^2\frac{A}{2}+\sum_{cyc}(bc(b^2-c^2)\cos A)+\sum_{cyc}a\sin(B-C)+\frac{1}{2}\sum_{cyc}\frac{\sin\frac{A}{2}}{\cos\frac{B}{2}\cos\frac{C}{2}}=1.$$

Proposed by Nguyen Van Canh-Vietnam

U.2362 a,b,c,d −sides, e,f −diagonals in a cyclic quadrilateral.

$$\text{If}\begin{cases}(ac)^5-(bd)^5=242\\(ac)^3+(bd)^3=28\end{cases},\text{ then }\max\{e^2,f^2\}\geq 4.$$

Proposed by Radu Diaconu-Romania

U.2363 a,b,c −sides, V −volume, F −total area in a rectangular parallelepiped.

If $V=\sqrt[5]{216n^3}, n>0$ then $\max\{a^7,b^7,c^7\}\geq nF$.

Proposed by Radu Diaconu-Romania

U.2364 If in acute ΔABC, $\frac{1}{16}\leq\cot A\cot B\cot C\leq\frac{\sqrt{3}}{9}$ then:

$$\sqrt[3]{\frac{s^2r^2}{R}}\leq\frac{2}{3}\left(\sum_{cyc}\sqrt[4]{s_a\cdot\cot A}\right)\left(\prod_{cyc}\sqrt[4]{s_a\cdot\cot A}\right)\leq R\sqrt[4]{\frac{s\sqrt{3}}{r}}.$$

Proposed by Radu Diaconu-Romania

U.2365 In acute ΔABC the following relationship holds:

$$\frac{\mu^2(A)}{4}+\tan\frac{\mu^2(B)}{4}+\csc\frac{\mu^2(C)}{4}+\frac{r}{2R}>1.$$

Proposed by Radu Diaconu-Romania

U.2366 Let $a,b,c\in[\alpha,\beta], 0\leq\alpha<\beta\leq 1, \alpha+\beta=1$. Find the maximum value of:

$$P=a+b+c-ab-bc-ca+2021.$$

Proposed by Nguyen Van Canh-Vietnam

U.2367 Find all functions $f:\mathbb{R}\to\mathbb{R}$ such that

$$f(\min\{x,y,0\})+f(\max\{x,y,0\})=2f(\min\{x,y,x+y,x-y\}),\forall x,y\in\mathbb{R}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2368 In ΔABC the following relationship holds:

$$\frac{a^4}{b^2+c^2}+\frac{b^4}{c^2+a^2}+\frac{c^4}{a^2+b^2}+\frac{R^4-16r^4}{r^2}\geq\frac{a^3}{b+c}+\frac{b^3}{c+a}+\frac{c^3}{a+b}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2369 In $\Delta ABC, \alpha \geq 5$, prove that: $\sum_{cyc} w_a^2 + \frac{\alpha s^2(R-2r)}{r} \geq \frac{R}{2r}\sum_{cyc} m_a^2$.

Proposed by Nguyen Van Canh-Vietnam

U.2370 In ΔABC the following relationship holds:

$$\sqrt{\sum_{cyc} \sin\frac{A}{2}\sin\frac{B}{2}} \leq \frac{3}{2}, \qquad \sqrt{\sum_{cyc} \cos\frac{A}{2}\cos\frac{B}{2}} \leq \frac{3\sqrt{3}}{2}, \qquad \sqrt[3]{\sum_{cyc} \sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}} \leq 1 + \frac{\sqrt{3}}{2}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2371 In $\Delta ABC, 2s = 3$. Prove that:

$$\sum_{cyc}\left(a\sqrt{b} + b\sqrt{a}\right) \leq 6 \leq \frac{2Rs^2}{9r^3}\left(\sum_{cyc}\frac{a^2}{b^2+c^2}\right)\cdot\frac{abc}{a^3+b^3+c^3}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2372 Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that: $2020f(x^2+y^2) + 2021f(x^2-y^2) = 2022f(xy), \forall x, y \in \mathbb{R}$.

Proposed by Nguyen Van Canh-Vietnam

U.2373 Find all functions $f: \mathbb{Z} \to \mathbb{Z}$ such that

$$5f(\min\{1,x,y\}) + 4f(\max\{0,x,y\}) = 3f(\max\{1,x,y\}) + 6f(\min\{0,x,y\}), \forall x, y \in \mathbb{Z}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2374 Solve for real numbers:

$$\log(\tan x) + \tan x = \log(\cot x) + \cot x, \sqrt{x^2-3x} \leq 6-5x, \qquad 2^x + 3^x = 4^x + 5^x.$$

Proposed by Nguyen Van Canh-Vietnam

U.2375 In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{a^2}{b^2+c^2} \geq \frac{3}{2} + \frac{r(R-2r)}{R^2+r^2}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2376 Find all positive real numbers α such that:

$$\sum_{cyc} m_a^2 + \alpha r(R-2r) \leq \sum_{cyc} n_a^2.$$

Proposed by Nguyen Van Canh-Vietnam

U.2377 Let $a, b, c \in \mathbb{R}$. Find a function $f(x) = |ax^2 + bx + c|\sqrt{1 - x^2}, |f(x) \leq 1, \forall |x| \leq 1$.

Proposed by Nguyen Van Canh-Vietnam

U.2378 Let $\alpha, \beta \geq 0$. Find all functions $f: \mathbb{R} \to \mathbb{R}$ such that:

$$\alpha x f(y) + \beta y f(x) = (\alpha + \beta)(x + y) f(x + y), \forall x, y \in \mathbb{R}.$$

Proposed by Nguyen Van Canh-Vietnam

U.2379 Let $1 < \alpha < \beta < \gamma, x, y \in \mathbb{R}$. Prove that:

$$\frac{|(x-\beta)(x-\gamma)|}{|(\alpha-\beta)(\alpha-\gamma)|} + \frac{|(x-\alpha)(x-\beta)|}{|(\gamma-\alpha)(\gamma-\beta)|} + \frac{|(x-\alpha)(x-\gamma)|}{|(\beta-\alpha)(\beta-\gamma)|} \geq \frac{\sin(\alpha^2+\beta^2+\gamma^2)}{\alpha^2+\beta^2+\gamma^2}.$$

$$\alpha \log \alpha + \beta \log \beta + \gamma \log \gamma \geq \frac{2\alpha+\beta}{3} \log \frac{2\alpha+\gamma}{3}.$$

Proposed by Nguyen Van Canh-Vietnam

All solutions for proposed problems can be finded on the http//:www.ssmrmh.ro which is the adress of Romanian Mathematical Magazine-Interactive Journal.

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