R M M

ROMANIAN MATHEMATICAL MAGAZINE
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Prove that in any triangle ABC

$$27(a^4+b^4+c^4)^2 \geq 256(m_a^6m_b^2+m_b^6m_c^2+m_c^6m_a^2)$$

Proposed by Nguyen Viet Hung – Hanoi – Vietnam

Solution by Kevin Soto Palacios – Huarmey – Peru

Probar en un triàngulo ABC

$$27(a^4+b^4+c^4)^2 \geq 256(m_a^6m_b^2+m_b^6m_c^2+m_c^6m_a^2)$$

LEMMA – VASILE CÎRTOAJE

Siendo x, y, z números R, se verifica la siguiente desigualdad conocida

$$(x^2+y^2+z^2)^2 \geq 3(x^3y+y^3z+z^3x) \quad \textit{(A)}$$

Realizamos las siguientes sustituciones

$$x=m_a^2, y=m_b^2, z=m_c^2$$

$$\Rightarrow 16x^2+16y^2+16z^2=16m_a^4+16m_b^4+16m_c^4=$$

$$=(2b^2+2c^2-a^2)^2+(2c^2+2a^2-b^2)^2+(2a^2+2b^2-c^2)^2$$

$$LHS=(4b^4+4c^4+a^4+8b^2c^2-4a^2b^2-4c^2a^2)+$$

$$+(4a^4+4c^4+b^4+8c^2a^2-4b^2c^2-4a^2b^2)+$$

$$+(4a^4+4b^4+c^4+8a^2b^2-4a^2c^2-4b^2c^2)=9a^4+9b^4+9c^4$$

$$\Rightarrow x^2+y^2+z^2=\frac{9}{16}(a^4+b^4+c^4)$$

Lo cual implica en (A)

$$\Rightarrow 27(a^4+b^4+c^4)^2 \geq 256(m_a^6m_b^2+m_b^6m_c^2+m_c^6m_a^2)$$

(LQQD)