SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

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**If** $x,y,z\in(0,\infty), xyz=1$ **then:**

$$x\left(x-3(y+z)\right)^2+\left(3x-(y+z)\right)^2(y+z)\geq 27$$

*Proposed by Daniel Sitaru – Romania*

*Solution 1 by Ear Bunpheng - Phnom Penh – Cambodia*

*Solution 2 by Myagmarsuren Yadamsuren-Darkhan-Mongolia*

*Solution 1 by Ear Bunpheng - Phnom Penh – Cambodia*

$$S=x\left(x-3(y+z)\right)^2+\left(3x-(y+z)\right)^2(y+z)\geq 27$$

***Since*** $x,y,z\in(0,+\infty)$

***By Cauchy:*** $x+y+z\geq 3\sqrt[3]{xyz}$

$$x+y+z\geq 3+(xyz-1)$$

$$\Rightarrow S\geq x\left(x-3(3-x)\right)^2+\left(3x-(3-x)\right)^2(y+z)$$

$$\geq x(4x-9)^2+(4x-3)^2(3-x)$$

***expand all the factor.***

$$\geq 81x-72x^2+16x^3+27-72x+4x^2-9x+24x^2-16x^3\geq 27$$

***(True)***

*Solution 2 by Myagmarsuren Yadamsuren-Darkhan-Mongolia*

$$x(x^2-6x\cdot(y+z)+9(y+z)^2)+$$

$$+(y+z)\cdot(9x^2-6x\cdot(y+z)+(y+z)^2)=$$

$$=x^3+3x^2\cdot(y+z)+3x(y+z)^2+(y+z)^3=$$

$$=(x+y+z)^3\geq\left(3\sqrt[3]{xyz}\right)^3=27$$