SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

Filiala Mehedinți - Mehedinți Branch
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In ΔABC:

WALKER'S INEQUALITY-1

$$3\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\right)\geq (a^2+b^2+c^2)\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$$

Proof by Kevin Soto Palacios-Huarmey-Peru

$$\Rightarrow 3\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\right)\geq \left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\right)+\left(\frac{b^2}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}\right)+3$$

$$\Rightarrow 2\left(\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\right)\geq \left(\frac{b^2}{a^2}+\frac{c^2}{b^2}+\frac{a^2}{c^2}\right)+3$$

Realizamos lo siguientes cambios de variables:

$$a^2=x+z\geq 0,\ b^2=y+z\geq 0,\ c^2=x+y\geq 0\ \Leftrightarrow (x,y,z)\geq 0$$

La desigualdad es equivalente:

$$\Rightarrow 2\left(\frac{x+z}{y+z}+\frac{y+z}{x+y}+\frac{x+y}{x+z}\right)\geq \left(\frac{y+z}{x+z}+\frac{x+y}{y+z}+\frac{x+z}{x+y}\right)+3$$

$$\Rightarrow 2(x+z)^2(x+y)+2(y+z)^2(x+z)+2(x+y)^2(y+z)\geq$$
$$\geq (y+z)^2(x+y)+(x+y)^2(x+z)+(x+z)^2(y+z)+3\prod(x+y)$$

$$\Rightarrow 2\sum(x^2+z^2+2xz)(x+y)\geq$$

$$\geq \sum(y^2+z^2+2yz)(x+y)+3\sum xy(x+y)+6xyz$$

$$\Rightarrow 2\sum(x^2+z^2)(x+y)+4\sum xz(x+y)\geq$$

$$\geq \sum (y^2+z^2)(x+y) + 2\sum yz(x+y) + 3\sum xy(x+y) + 6xyz$$

$$\Rightarrow 2\sum (x^2+z^2)(x+y) + 4\sum x^2z + 12xyz \geq$$

$$\geq \sum (y^2+z^2)(x+y) + 2\sum y^2z + 3\sum xy(x+y) + 12xyz$$

$$2\sum (x^2+z^2)(x+y) = 2\sum x^3 + 4xz^2 + 4zy^2 + 4yx^2 + 2yz^2 + 2y^2x + 2x^2z$$

$$\sum (y^2+z^2)(x+y) = \sum x^3 + 2xy^2 + 2yz^2 + 2zx^2 + xz^2 + yx^2 + zy^2$$

$$3\sum xy(x+y) = 3x^2y + 3y^2x + 3y^2z + 3z^2y + 3z^2x + 3xz^2$$

$$4\sum x^2z = 4x^2z + 4z^2y + 4y^2x$$

$$2\sum y^2z = 2y^2z + 2x^2y + 2z^2x$$

$$\Rightarrow 2\sum (x^2+z^2)(x+y) - \sum (y^2+z^2)(x+y) =$$
$$= \sum x^3 + 3xz^2 + 3zy^2 + 3yx^2$$

$$\Rightarrow \sum x^3 + 3xz^2 + 3zy^2 + 3yx^2 - 2\sum y^2z = \sum x^3 + xz^2 + zy^2 + yx^2$$

La desigualdad es equivalente:

$$\Rightarrow \sum x^3 + xz^2 + zy^2 + yx^2 + 4x^2z + 4z^2y + 4y^2x \geq 3\sum xy(x+y)$$

$$\Rightarrow \sum x^3 - 2z^2x - 2y^2z - 2x^2y + x^2z + z^2y + y^2x \geq 0$$

$$\Rightarrow (x^3 - 2x^2y + y^2x) + (y^3 - 2y^2z + z^2y) + (z^3 - 2z^2x + x^2z) \geq 0$$

$$\Rightarrow x(x-y)^2 + y(y-z)^2 + z(z-x)^2 \geq 0$$ **(LQQD)**