SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

Filiala Mehedinți - Mehedinți Branch
www.ssmrmh.ro

**JP.025. Let $n \geq 2$ be an integer and let $a, b, c$ be positive numbers such that $ab + bc + ca \leq 1$. Prove that:**

$$\frac{bc}{(2a^2 + bc)^n} + \frac{ca}{(2b^2 + ca)^n} + \frac{ab}{(2c^2 + ab)^n} \geq 1$$

*Proposed by Nguyen Viet Hung – Hanoi – Vietnam*

*Solution by Kevin Soto Palacios – Huarmey – Peru*

**Sea: $n \geq 2$ un entero $y$ sean "$a, b, c$"$\mathbb{R}^+$, tal que: $ab + bc + ac \leq 1$. Probar que:**

$$\frac{bc}{(2a^2 + bc)^n} + \frac{ac}{(2b^2 + ca)^n} + \frac{ab}{(2c^2 + ab)^n} \geq \frac{1}{3}$$

**Realizando la desigualdad ponderada "Jensen" para:**

$$f(x) = \frac{1}{x^n}, x > 0, n \geq 2 \Leftrightarrow \textbf{(Convexo)}$$

$$\frac{bc}{(2a^2 + bc)^n} + \frac{ac}{(2b^2 + ca)^n} + \frac{ab}{(2c^2 + ab)^n} = bcf(bc + 2a^2) + acf(ac + 2b^2) + abf(ab + 2c^2)$$

$$bcf(bc + 2a^2) + acf(ac + 2b^2) + abf(ab + 2c^2) \geq$$
$$\geq (bc + ac + ab)f\left(\frac{bc(bc + 2a^2) + ac(ac + 2b^2) + ab(ab + 2c^2)}{ab + bc + ac}\right)$$

$$(bc + ac + ab)f\left(\frac{(bc)^2 + (ac)^2 + (ab)^2 + 2abc(a + b + c)}{ab + bc + ac}\right) = (bc + ac + ab)f\left(\frac{(ab + bc + ac)^2}{ab + bc + ac}\right)$$

$$(bc + ac + ab)f\left(\frac{(ab + bc + ac)^2}{ab + bc + ac}\right) = (bc + ac + ab)f(ab + bc + ac) = \frac{1}{(ab + bc + ac)^{n-1}} \geq 1$$