SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

Filiala Mehedinți - Mehedinți Branch
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If $0 \le x \le 3,\ 0 \le y \le 4,\ a > 0$

$$\Omega_1 = \int_0^a \left(\int_0^a \sqrt{x^2 + y^2 - 6x + 9}\, dx \right) dy$$

$$\Omega_2 = \int_0^a \left(\int_0^a \sqrt{x^2 + y^2 - 8y + 16}\, dx \right) dy$$

then: $\Omega_1 + \Omega_2 > 5a^2$

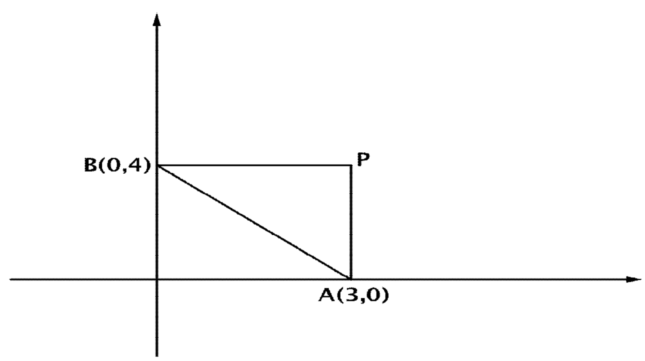
Proposed by Daniel Sitaru – Romania

Solution 1 by Ravi Prakash - New Delhi – India

$$PA + PB = \sqrt{x^2 + y^2 - 6x + 9} + \sqrt{x^2 + y^2 - 8y + 16}$$

$$= \sqrt{(x-3)^2 + y^2} + \sqrt{(x^2) + (y-4)^2}$$

$$\ge \sqrt{3^2 + y^2} = 5$$



$$\Omega_1 + \Omega_2 = \int_0^a \left(\int_0^a \left(\sqrt{x^2 + y^2 - 6x + 9} + \sqrt{x^2 + y^2 - 8y + 16} \right) dx \right) dy \ge$$

$$\geq \int_0^a \left(\int_0^a 5dx \right) dy = 5a^2$$

Solution 2 by SK Rejuan – West Bengal – India

$$0 \leq x \leq 3, 0 \leq y \leq 9, a > 0$$

$$\Omega_1 = \int_0^a \left(\int_0^a \sqrt{x^2 + y^2 - 6x + 9}\, dx \right) dy = \int_0^a \left(\int_0^a \sqrt{(3 - x)^2 + y^2}\, dx \right) dy \geq$$

$$\geq \int_0^a \left\{ \int_0^a \frac{1}{\sqrt{2}} (3 - x + y) dx \right\} dy = \frac{1}{\sqrt{2}} \int_0^a \left[\int_0^a \{(3 + y) - x\} dx \right] dy =$$

$$= \frac{1}{\sqrt{2}} \int_0^a \left\{ \left[(3 + y)x - \frac{x^2}{2} \right]_0^a \right\} dy = \frac{1}{\sqrt{2}} \int_0^a \left\{ (3 + y)a - \frac{a^2}{2} \right\} dy =$$

$$= \frac{1}{\sqrt{2}} \left[3a + \frac{y^2}{2} a - \frac{a^2}{2} y \right]_0^a = \frac{1}{\sqrt{2}} \left(3a^2 + \frac{a^3}{2} - \frac{a^2}{2} \right) = \frac{3}{\sqrt{2}} a^2$$

$$\Rightarrow \Omega_1 \geq \frac{3}{\sqrt{2}} a^2 \quad \text{(1)}$$

$$\Omega_2 = \int_0^a \left(\int_0^a \sqrt{x^2 + y^2 - 8y + 16}\, dx \right) dy = \int_0^a \left(\int_0^a \sqrt{x^2 + (4 - y)^2} dx \right) dy =$$

$$\geq \int_0^a \left\{ \int_0^a \frac{1}{\sqrt{2}} (x + 4 - y) dx \right\} dy$$

$$\Rightarrow \Omega_2 \geq \frac{1}{\sqrt{2}}\int_0^a \left[\int_0^a \{x + (4 - y)\}\, dx\right] dy = \frac{1}{\sqrt{2}}\int_0^a \left\{\left[\frac{x^2}{2} + (4 - y)x\right]_0^a\right\} dy =$$

$$= \frac{1}{\sqrt{2}}\int_0^a \left\{\frac{a^2}{2} + (4 - y)a\right\} dy = \frac{1}{\sqrt{2}}\left[\frac{a^2}{2}y + (4ay) - \frac{y^2}{2}a\right]_0^a =$$

$$= \frac{1}{\sqrt{2}}\left(\frac{a^2}{2} + 4a^2 - \frac{a^3}{2}\right) = 2\sqrt{2}a^2$$

$$\Rightarrow \Omega_2 \geq 2\sqrt{2}a^2 \qquad (2)$$

$$\Omega_1 + \Omega_2 \geq \frac{3}{\sqrt{2}}a^2 + 2\sqrt{2}a^2 = \left(\frac{3 + 9}{\sqrt{2}}\right)a^2$$

$$\Rightarrow \Omega_1 + \Omega_2 \geq \frac{7}{\sqrt{2}}a^2 = \left(\frac{7}{2}\right)\sqrt{2}a^2$$

$$\Rightarrow \Omega_1 + \Omega_2 \geq 3 \cdot 5\sqrt{2}a^2 > 5a^2$$

$$\Rightarrow \Omega_1 + \Omega_2 > 5a^2$$