If $0 \le x \le a, 0 \le y \le b, c > 0$

$$\Omega_1 = \int_0^c \left(\int_0^c \sqrt{x^2 + y^2 - 2ax + a^2}\, dx \right) dy$$

$$\Omega_2 = \int_0^c \left(\int_0^c \sqrt{x^2 + y^2 - 2by + b^2} dx \right) dy$$

then $\Omega_1 + \Omega_2 \le (a + b)c^2$

Proposed by Daniel Sitaru – Romania

Solution by Subhajit Chattopadhyay-Bolpur-India

$$\Omega_1 + \Omega_2 = \int_0^c \int_0^c \left(\sqrt{x^2 + y^2 - 2ax + a^2} + \sqrt{x^2 + y^2 - 2by + b^2} \right) dx\, dy$$

Now, $\sqrt{x^2 + y^2 - 2ax + a^2} + \sqrt{x^2 + y^2 - 2by + b^2}$

$$= \sqrt{(a - x)^2 + y^2} + \sqrt{(b - y)^2 + x^2}$$

Again, $\sqrt{(a - x)^2 + y^2} \le a - x + y$

$$[0 \le x \le a]$$

and $\sqrt{(b - y)^2 + x^2} \le b - y + x$

Hence, $\sqrt{(a - x)^2 + y^2} + \sqrt{(b - y)^2 + x^2} \le a - x + y + b - y + x = a + b$

$$\Omega_1 + \Omega_2 \le \int_0^c \int_0^c (a+b)\, dx\, dy = (a+b)c^2$$