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If  $0 \leq x \leq a, 0 \leq y \leq b, c > 0$

$$\Omega_1 = \int_0^c \left( \int_0^c \sqrt{x^2 + y^2 - 2ax + a^2} dx \right) dy$$

$$\Omega_2 = \int_0^c \left( \int_0^c \sqrt{x^2 + y^2 - 2by + b^2} dx \right) dy$$

then  $\Omega_1 + \Omega_2 \leq (a + b)c^2$

*Proposed by Daniel Sitaru – Romania*

*Solution by Subhajit Chattopadhyay-Bolpur-India*

$$\Omega_1 + \Omega_2 = \int_0^c \int_0^c \left( \sqrt{x^2 + y^2 - 2ax + a^2} + \sqrt{x^2 + y^2 - 2by + b^2} \right) dx dy$$

$$\text{Now, } \sqrt{x^2 + y^2 - 2ax + a^2} + \sqrt{x^2 + y^2 - 2by + b^2}$$

$$= \sqrt{(a-x)^2 + y^2} + \sqrt{(b-y)^2 + x^2}$$

$$\text{Again, } \sqrt{(a-x)^2 + y^2} \leq a - x + y$$

$$[0 \leq x \leq a]$$

$$\text{and } \sqrt{(b-y)^2 + x^2} \leq b - y + x$$

$$\text{Hence, } \sqrt{(a-x)^2 + y^2} + \sqrt{(b-y)^2 + x^2} \leq a - x + y + b - y + x = a + b$$

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$$\Omega_1 + \Omega_2 \leq \int_0^c \int_0^c (a + b) dx dy = (a + b)c^2$$