SOCIETATEA DE ȘTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

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If $f,g:[a,b]\to(0,\infty)$ integrable, such that $f(x)+g(x)\le 8$ then:

$$\int_a^b \frac{f(x)\sqrt{g(x)}+g(x)\sqrt{f(x)}}{f(x)-\sqrt{f(x)g(x)}+g(x)}dx\le 4(b-a)$$

Proposed by Daniel Sitaru – Romania

Solution by Anas Adlany - El Jadida – Morroco

We have for all $t,z>0$;

$$\frac{t\sqrt{z}+z\sqrt{t}}{t+z-\sqrt{zt}}=\sqrt{zt}\cdot\frac{\sqrt{z}+\sqrt{t}}{\underbrace{t+z-\sqrt{zt}}_{\ge\sqrt{zt}}}\le\sqrt{z}+\sqrt{t}\le\sqrt{2(z+t)}$$

(because $x+y\le\sqrt{2}\sqrt{x^2+y^2}$)

So put $z=f(x)$ and $t=g(x)$ to find:

$$\frac{f(x)\sqrt{g(x)}+g(x)\sqrt{f(x)}}{f(x)-\sqrt{f(x)g(x)}+g(x)}\le\sqrt{2}\sqrt{\big(f(x)+g(x)\big)}\le 4\Rightarrow$$

$$\Rightarrow\int_a^b \frac{f(x)\sqrt{g(x)}+g(x)\sqrt{f(x)}}{f(x)-\sqrt{f(x)g(x)}+g(x)}dx\le 4(b-a)$$