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In ΔABC the following relationship holds:

$$3 \left(\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} \right) \geq \sum \frac{b^2 + bc + c^2}{bcm_a}$$

Proposed by Daniel Sitaru – Romania

Solution by Soumava Chakraborty – Kolkata – India

Using Tereshin's inequality,

$$m_a \geq \frac{b^2 + c^2}{4R},$$

$$m_b \geq \frac{c^2 + a^2}{4R},$$

$$m_c \geq \frac{a^2 + b^2}{4R}$$

$$\sum \frac{b^2 + bc + c^2}{bcm_a} \leq 4R \left\{ \frac{b^2 + c^2 + bc}{bc(b^2 + c^2)} + \frac{c^2 + a^2 + ca}{ca(c^2 + a^2)} + \frac{a^2 + b^2 + ab}{ab(a^2 + b^2)} \right\} =$$

$$= 4R \left\{ \left(\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} \right) + \left(\frac{1}{b^2 + c^2} + \frac{1}{c^2 + a^2} + \frac{1}{a^2 + b^2} \right) \right\} =$$

$$\leq 4R \left\{ \frac{a + b + c}{abc} + \frac{1}{2} \left(\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} \right) \right\} =$$

$$(b^2 + c^2 \geq 2bc, c^2 + a^2 \geq 2ca, a^2 + b^2 \geq 2ab)$$

$$= 4R \left(\frac{a+b+c}{abc} + \frac{a+b+c}{2abc} \right) = 4R \cdot \frac{3}{2} \left(\frac{2s}{4Rrs} \right) = \frac{3}{r} \quad (1)$$

Now,

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$$\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} = 2R \left(\frac{1}{bc} + \frac{1}{ca} + \frac{1}{ab} \right) = 2R \frac{2s}{abc} = \frac{4Rs}{4Rrs} = \frac{1}{r}$$

$$3 \left(\frac{1}{h_a} + \frac{1}{h_b} + \frac{1}{h_c} \right) = \frac{3}{r} \quad (2)$$

(1) and (2) \Rightarrow the desired inequality