



Prove that in any triangle ABC :

$$\frac{r_a}{(s-b)(s-c)} + \frac{r_b}{(s-c)(s-a)} + \frac{r_c}{(s-a)(s-b)} \geq \frac{2}{r} \sqrt{\frac{4R+r}{2R}}$$

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Probar en en triángulo ABC , la siguiente desigualdad:

$$\frac{r_a}{(s-b)(s-c)} + \frac{r_b}{(s-a)(s-c)} + \frac{r_c}{(s-b)(s-a)} \geq \frac{2}{r} \sqrt{\frac{4R+r}{2R}}$$

Recordar lo siguiente en un triángulo ABC :

$$r_a = \frac{S}{s-a}, r_b = \frac{S}{s-b}, r_c = \frac{S}{s-c}, S = \frac{abc}{4R}$$

$$r = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = 4R \frac{(s-a)(s-b)(s-c)}{abc}$$

Reemplazando en la desigualdad...

$$\begin{aligned} & \frac{S}{(s-a)(s-b)(s-c)} + \frac{S}{(s-a)(s-b)(s-c)} + \frac{S}{(s-a)(s-b)(s-c)} \geq \\ & \geq \frac{2abc}{4R(s-a)(s-b)(s-c)} \sqrt{\frac{4R+r}{2R}} \end{aligned}$$

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$$\Rightarrow \frac{6SR}{abc} \geq \sqrt{\frac{4R+r}{2R}} \rightarrow \frac{3}{2} \geq \sqrt{\frac{4R+r}{2R}} \rightarrow \frac{9}{4} \geq 2 + \frac{r}{2R} \rightarrow \frac{1}{4} \geq \frac{r}{2R} \Leftrightarrow$$

$\Leftrightarrow R \geq 2r \dots$ (Desigualdad de Euler)