In ΔABC **the following relationship holds:**

$$\sum \frac{r_a}{r_b}\left(1-\frac{r}{r_c}\right) \geq 2$$

Proposed by Nicolae Nica - Romania

Solution 1 by Adil Abdullayev – Baku – Azerbaidian

$$LHS = \frac{r_a}{r_b}+\frac{r_b}{r_c}+\frac{r_c}{r_a}-\frac{(4R+r)^2}{p^2}+2 =$$

$$= \frac{r_a^2}{r_a r_b}+\frac{r_b^2}{r_c r_b}+\frac{r_c^2}{r_a r_c}-\frac{(4R+r)^2}{p^2}+2 \overset{C-B-S}{\geq}$$

$$\overset{C-B-S}{\geq} \frac{(4R+r)^2}{p^2}-\frac{(4R+r)^2}{p^2}+2 = 2.$$

Solution 2 by Daniel Sitaru – Romania

$$\sum \frac{r_a}{r_b}\left(1-\frac{r}{r_c}\right) = \sum \frac{s-b}{s-a}\left(1-\frac{s-c}{s}\right) = \frac{1}{s}\sum \frac{(s-b)c}{s-a} =$$

$$= \frac{1}{s}\sum \frac{y(x+y)}{x} \geq 2 \leftrightarrow \sum \frac{y(x+y)}{x} \geq 2(x+y+z) \leftrightarrow$$

$$(a = y+z, b = z+x, c = x+y)$$

$$\leftrightarrow \frac{yz}{x}+\frac{xy}{z}+\frac{xz}{y} \geq x+y+z \leftrightarrow$$

$$\leftrightarrow x^2y^2+y^2z^2+z^2x^2 \geq xyz(x+y+z)$$

$$(2,2,0) \succcurlyeq (2,1,1) \text{(Muirhead)}$$