SOCIETATEA DE ŞTIINȚE MATEMATICE DIN ROMÂNIA
ROMANIAN MATHEMATICAL SOCIETY

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Let a, b, c, d be non-negative real numbers. Prove that:

$$\sqrt{(a^2+b^2)(c^2+d^2)}+2\sqrt{abcd}\geq (a+b)\sqrt{cd}+(c+d)\sqrt{ab}$$

Proposed by Nguyen Viet Hung – Hanoi – Vietnam

Solution by Kevin Soto Palacios – Huarmey – Peru

Sean: a, b, c, d números $\mathbb{R}$ no negativos. Probar la siguiente desigualdad:

$$\sqrt{(a^2+b^2)(c^2+d^2)}+2abcd\geq (a+b)\sqrt{cd}+(c+d)\sqrt{ab}$$

Por: $MC \geq MA$

$$\sqrt{(a^2+b^2)(c^2+d^2)}+2abcd\geq \frac{(a+b)(c+d)}{2}+2abcd\geq (a+b)\sqrt{cd}+(c+d)\sqrt{ab}$$

Sean: $a=x^2\geq 0, b=y^2\geq 0, c=z^2\geq 0, d=w^2\geq 0$

La desigualdad es equivalente:

$$(a+b)(c+d)+4abcd\geq 2(a+b)\sqrt{cd}+2(c+d)\sqrt{ab}$$

$$(x^2+y^2)(z^2+w^2)+4xyzw\geq 2x^2zw+2y^2zw+2z^2xy+2w^2xy$$
$$(xz)^2+(xw)^2+(yw)^2+(yz)^2+4xyzw$$
$$\geq 2x^2zw+2y^2zw+2z^2xy+2w^2xy$$

$$\Rightarrow x^2(z^2+w^2-2wz)+y^2(z^2+w^2-2wz)-2xy(z^2+w^2-2wz)\geq 0$$

$$\Leftrightarrow (z-w)^2(x^2+y^2-2xy)=(z-w)^2(x-y)^2\geq 0 \rightarrow$$ **(Lo cual es cierto)**